The Stoic Syllogism through Two Tropics

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Origen, Against Celsus VII.15

He assumes that certain things are impossible and improper for God to do, saying: If these things were prophesied about the supreme God, ought we then to believe such things about God because they are predicted? And he thinks he can argue that even if it is really true that the prophets foretold such things about God's son, it would be impossible to believe in the predictions that he should suffer and do these things. We may reply that his assumption is mistaken and would make hypothetical premisses result in contradictory conclusions. This is shown as follows: (i) If the prophets of the supreme God were to say that God will serve as a slave or will be sick or even that He will die, these things will happen to God, since the prophets of the great God must necessarily speak the truth, (ii) On the other hand, if the true prophets of the supreme God say these same things, since things that are intrinsically impossible are not true, what the prophets say of God would not happen. But when two conditional premisses result in contradictory conclusions by what is known as the theorem through two tropics, the antecedent of the two premisses is denied, which in this instance is that the prophets foretell that the great God will serve as a slave or will be sick or will die. The conclusion is therefore that the prophets did not foretell that the great God will be a slave or will be sick or will die. The argument runs like this: If the first, then the second; if the first, then not the second; so, not the first.

The Stoics give the following concrete illustration of this when they say: If you know that you are dead, you are dead; if you know that you are dead you are not dead; it follows that you do not know that you are dead. This is the way in which they make up the premisses. If you know that you are dead, what you know is true; then it is true that you are dead. And on the other hand, if you know that you are dead, then it is also true that you are dead, you are dead. But since a dead man knows nothing, obviously if you know that you are dead, you are not dead. And as I said before, it follows from both premisses that you do not know that you are dead. The same sort of argument is implicit in Celsus' assumption when he makes the remark we have quoted. (Chadwick trans, heavily modified)

Έπεὶ δ' ἀδύνατά τινα καὶ ἀπρεπῆ θεῷ καθ' ὑπόθεσιν τιθείς φησιν· Εἰ ταῦτα προφητεύοιτο περὶ τοῦ ἐπὶ πᾶσι θεοῦ, ἄρ' ἐπεὶ προλέγεται, πιστεύεσθαι δεῖ τὰ τοιαῦτα περὶ θεοῦ; Καὶ νομίζει κατασκευάζεσθαι ὅτι, κἂν ἀληθῶς ῶσι προειρηκότες οἱ προφῆται περὶ υἱοῦ θεοῦ τοιαῦτα, ἀδύνατον ἤν αὐτὸν παθεῖν <οῦτως> ἢ δρᾶσαι χρῆναι πιστεύειν τοῖς προειρημένοις· λεκτέον ὅτι ἡ ὑπόθεσις αὐτοῦ ἄτοπος οῦσα ποιήσαι ἂν συνημμένα εἰς τὰ ἀλλήλοις ἀντικείμενα λήγοντα, ὅπερ οὕτω δείκνυται. Εἰ οἱ ἀληθῶς προφῆται τοῦ ἐπὶ πᾶσι θεοῦ δουλεύειν ἢ νοσεῖν ἢ καὶ τεθνήξεσθαι φήσαιεν τὸν θεόν, συμβήσεται ταῦτα περὶ τὸν θεόν, ἀψευδεῖν γὰρ ἀνάγκη τοὺς τοῦ μεγάλου θεοῦ προφήτας· ἀλλὰ καὶ εἰ οἱ ἀληθῶς προφῆται τοῦ ἐπὶ πᾶσι θεοῦ τὰ αὐτὰ ταῦτά φασιν, ἐπεὶ τὰ τῆ φύσει ἀδύνατα οὐκ ἔστιν ἀληθῆ, οὐκ ἂν συμβαίη περὶ τὸν θεὸν ὰ λέγουσιν οἱ προφῆται. Ὅταν δὲ δύο συνημμένα λήγη εἰς τὰ ἀλλήλοις ἀντικείμενα λήγη εἰς τὰ ἀλλήλοις ἀνομβαίη περὶ τὸν θεὸν ἀ

τροπικῶν θεωρήματι, ἀναιρεῖται τὸ ἐν ἀμφοτέροις τοῖς συνημμένοις ἡγούμενον, ὅπερ ἐν τούτοις ἐστὶ τὸ προλέγειν τοὺς προφήτας τὸν μέγαν θεὸν δουλεύειν ἢ νοσήσειν ἢ τεθνήξεσθαι. Συνάγεται οῦν τὸ οὐκ ἀρα προεῖπον οἱ προφῆται τὸν μέγαν θεὸν δουλεύειν ἢ νοσήσειν ἢ τεθνήξεσθαι, καὶ ὑπάγεταί γε ὁ λόγος τρόπῳ τοιούτῳ· εἰ τὸ πρῶτον, καὶ τὸ δεύτερον· εἰ τὸ πρῶτον, οὐ τὸ δεύτερον· οὐκ ἄρα τὸ πρῶτον.

Φέρουσι δὲ καὶ ἐπὶ ὕλης τὸν τρόπον τοῦτον οἱ ἀπὸ τῆς Στοᾶς, λέγοντες τό· εἰ ἐπίστασαι ὅτι τέθνηκας, < <τέθνηκας· εἰ ἐπίστασαι ὅτι τέθνηκας,> οὐ τέθνηκας· ἀκολουθεῖ τὸ οὐκ ἄρα ἐπίστασαι ὅτι τέθνηκας. Τὸν τρόπον δὲ τοῦτον κατασκευάζουσι τὰ συνημμένα· εἰ ἐπίστασαι ὅτι τέθνηκας, ἔστιν ὃ ἐπίστασαι, ἔστιν ἄρα τὸ τέθνηκας. Καὶ πάλιν· εἰ ἐπίστασαι ὅτι τέθνηκας, καὶ ἔστιν τὸ ἐπίστασαι ὅτι τέθνηκας. Ἐπεὶ δὲ ὁ τεθνηκώς <οὐδὲν> ἐπίσταται, δῆλον ὅτι, εἰ ἐπίστασαι ὅτι ἑ τεθνηκώς <οὐδὲν> ἐπίσταται, δῆλον ὅτι, εἰ ἐπίστασαι ὅτι τέθνηκας, οὐ τέθνηκας. Καὶ ἀκολουθεῖ, ὡς προεῖπον, ἀμφοτέροις τοῖς συνημμένοις τὸ οὐκ ἄρα ἐπίστασαι ὅτι τέθνηκας, οὐ τέθνηκας. Καὶ ἀκολουθεῖ, ὡς προεῖπον, ἀμφοτέροις τοῖς συνημμένοις τὸ οὐκ ἀρα ἐπίστασαι ὅτι τέθνηκας. Τοιοῦτόν τι ἐστὶ καὶ περὶ τὴν Κέλσου ὑπόθεσιν, λέγοντος ἡν προεξεθέμεθα λέξιν.

Galen, On the Doctrines of Hippocrates and Plato II.3.18-19

Now you can meet many people minutely skilled in the ways of analyzing syllogisms through two or three tropics, indifferently concluding syllogisms, or certain others of this kind, which use the first and second ground-rules, as also in connexion with other syllogisms which they analyze by means of the third or fourth ground-rule. Yet most of these can be analysed in another, more succinct way, as Antipater wrote. And anyway, all the construction of such syllogisms is no small overexpenditure of effort on something useless, as Chrysippus himself testifies in practice by never in his own works needing those syllogisms to demonstrate a doctrine. (Long and Sedley trans, lightly modified)

νυνὶ δὲ πῶς μὲν οἱ διὰ δύο τροπικῶν <ἢ> τριῶν ἀναλύονται συλλογισμοὶ καὶ πῶς οἱ ἀδιαφόρως περαίνοντες ἤ τινες ἄλλοι τοιοῦτοι τῷ πρώτῷ καὶ δευτέρῷ θέματι προσχρώμενοι, πολλοῖς ἐστι συντυχεῖν ἀκριβῶς ἠσκημένοις, ὥσπερ ἀμέλει καὶ ἐπ΄ ἄλλοις ὅσους διὰ τοῦ τρίτου θέματος ἢ τετάρτου συλλογισμοὺς ἀναλύουσι. καίτοι τούτων τοὺς πλείστους ἔνεστιν ἑτέρως ἀναλύειν συντομώτερον, ὡς Ἀντίπατρος ἔγραψε, πρὸς τῷ καὶ περιεργίαν εἶναι οὐ μικρὰν ἀχρήστου πράγματος ἅπασαν τὴν τῶν τοιούτων συλλογισμῶν πλοκήν, ὡς αὐτὸς ὁ Χρύσιππος ἔργῷ μαρτυρεῖ μηδαμόθι τῶν ἑαυτοῦ συγγραμμάτων εἰς ἀπόδειξιν δόγματος ἐκείνων δεηθεὶς τῶν συλλογισμῶν.

Sextus Empiricus, Outlines of Pyrrhonism II.2-3

They say that Sceptics either apprehend what the Dogmatists talk about or do not apprehend it. If they apprehend it, how can they be puzzled about what they say they apprehend? If they do not apprehend it, they do not even know how to talk about what they have not apprehended. For just as someone who does not know what, for example, the removal argument or the theorem in two tropics is cannot even say anything about them, so someone who does not recognize any of the items the Dogmatists talk about cannot conduct an investigation in opposition to them about things which he does not know. In neither case, therefore, can Sceptics investigate what the Dogmatists talk about. (Annas and Barnes trans., lightly modified)

φασὶ γὰρ ὡς ἦτοι καταλαμβάνει ὁ σκεπτικὸς τὰ ὑπὸ τῶν δογματικῶν λεγόμενα ἢ οὐ καταλαμβάνει· καὶ εἰ μὲν καταλαμβάνει, πῶς ἂν ἀποροίη περὶ ῶν κατειληφέναι λέγει; εἰ δ' οὐ καταλαμβάνει, ἄρα περὶ ῶν οὐ κατείληφεν οὐδὲ οἴδε λέγειν. ὥσπερ γὰρ ὁ μὴ εἰδώς, εἰ τύχοι, τί ἐστι τὸ καθ' ὃ περιαιρουμένου ἢ τὸ διὰ δύο τροπικῶν θεώρημα, οὐδὲ εἰπεῖν τι δύναται περὶ αὐτῶν, οὕτως ὁ μὴ γινώσκων ἕκαστον τῶν λεγομένων παρὰ τοῖς δογματικοῖς οὐ δύναται ζητεῖν πρὸς αὐτοὺς περὶ ῶν οὐκ οἶδεν. οὐδαμῶς ἄρα δύναται ζητεῖν ὁ σκεπτικὸς περὶ τῶν λεγομένων παρὰ τοῖς δογματικοῖς.

Sextus Empiricus, Outlines of Pyrrhonism II.189

But it is impossible, according to them [the Stoics] for a conditional composed of conflicting statements to be sound. For a conditional announces that if its antecedent is the case, then so too is its consequent, and conflicting statements announce the contrary—that if either one of them is the case, it is impossible for the other to hold. If, therefore, this conditional— 'if there are proofs, there are proofs' is sound, then the conditional— 'if there are not proofs, there are proofs' cannot be sound. (Annas and Barnes trans.)

ἀδύνατον δέ ἐστι κατ' αὐτοὺς συνημμένον ὑγιὲς εἶναι ἐκ μαχομένων ἀξιωμάτων συνεστώς. τὸ μὲν γὰρ συνημμένον ἐπαγγέλλεται ὄντος τοῦ ἐν αὐτῷ ἡγουμένου εἶναι καὶ τὸ λῆγον, τὰ δὲ μαχόμενα τοὐναντίον, ὄντος τοῦ ἑτέρου αὐτῶν ὁποιουδήποτε ἀδύνατον εἶναι τὸ λοιπὸν ὑπάρχειν. ὄντος ἄρα ὑγιοῦς τοῦδε τοῦ συνημμένου 'εἰ ἔστιν ἀπόδειξις, ἔστιν ἀπόδειξις' οὐ δύναται ὑγιὲς εἶναι τοῦτο <τὸ> συνημμένον 'εἰ οὐκ ἔστιν ἀπόδειξις, ἔστιν ἀπόδειξις.'

Aulus Gellius, Attic Nights 16.8.12-14

There is likewise another, which the Greeks call *diezeugmenon axioma* and we call a "disjunctive" proposition. It is of this form: "Either pleasure is bad, or it is good, or it is neither good nor bad." But all the disjuncts must mutually conflict, and their contradicotries (which the Greeks call *antikeimena*) must also be mutually opposed. Of all the disjuncts, one must be true, the others false. But if either none of them is true, or all or more than one of them are true, or the disjuncts do not conflict, or the contradictories of the disjuncts are not mutually incompatible, then that is false as a disjunctive proposition... (Long and Sedley, trans.)

est item aliud, quod Graeci διεζευγμένον ἀξίωμα, nos "disiunctum" dicimus. id huiuscemodi est: "aut malam est voluptas aut bonum aut neque bonum neque malum est." omnia autem, quae disiunguntur, pugnantia esse inter sese oportet, eorumque opposita, quae ἀντικείμενα Graeci dicunt, ea quoque ipsa inter se adversa esse. ex omnibus, quae disiunguntur, unum esse verum debet, falsa cetera. quod si aut nihil omnium verum, aut omnia plurave quam unum vera erunt, aut quae disiuncta sunt non pugnabunt, aut quae opposita eorum sunt contraria inter sese non erunt, tunc id disiunctum mendacium est...