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WORLDS AWAY:
VALIDITY IN JOHN BURIDAN'S MODAL LOGIC

ABSTRACT: In this talk, I set out Buridan's semantics for divided (*de re*) modal propositions, and I conclude that, contrary to several claims in the literature, Buridan's modal logic *cannot* be accommodated on any Kripke-style possible worlds semantics.

In part one I sketch Buridan's semantics for assertoric propositions, on which he bases his semantics for modals. In part two, I outline ampliation (*ampliatio*) in modal contexts. In Buridan's system, modal propositions are characterised by the ampliation of their terms, which are extended to *possibilia* as well as actual things. In part three, I discuss two axioms Buridan bases on his semantics, both of which deal with the relations among classes of modals and their assertoric counterparts. In section four, after a brief overview of the modern method of analysing modal propositions in terms of possible worlds, I show why these two axioms cannot be accommodated on any consistent Kripke-style class of frames. Buridan's modal logic is, then, not equivalent to any normal modern modal system.

I. ampliation, or, how to make a proposition modal

First, a word on propositions and what makes them true:

[text 1] For the truth of an affirmative categorical proposition, it is necessary that the terms stand for the same thing.[1]

[text 2] Whatever and however many things are required for the truth of an affirmative proposition, as far as the things signified are concerned, a failure of one of them is enough for its negative contradictory to be true, since otherwise wouldn't be necessary that if one were true, the other would be false, and vice-versa.[2]

[text 3] For the truth of a negative proposition, it is enough that the terms do not stand for the same thing.[3]

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[1] "Ad veritatem affirmativae et categoricae requiritur [...] quod termini supponant pro eadem re." (*In Metaphysicen Aristotelis Quaestiones* (*MAQ*) VI.7 fol. 38v, b).
[2] "Quaecumque et quot requiruntur ad veritatem affirmativae quantum est ex parte rerum significatarum, defectus unius illarum sufficit ad veritatem negativae contradictoriae, quia aliter non esset necesse si una esset vera quod altera esset falsa, et econverso. (*MAQ* VI.7, fol.38vb-c).
[3] "Ad veritatem negativae sufficit quod termini non supponant pro eodem." (*MAQ* VI.7, fol.38vc).

Note that from text 3, and the fact that affirmative propositions have existential import, it follows that E- and I-type propositions do not. And Buridan explicitly acknowledges this.[4] These facts about existential import will become important in section III.ii, below.

i. the four basic types of proposition

A: Universal affirmative: subject's extension (a) is included in the predicate's (b):

E.g. "Every human (a) is an animal (b)" (a → b)

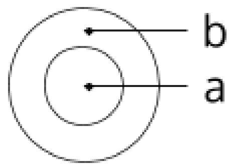

Figure 1: the extension of a *is contained in the extension of* b.

E: Universal negative: subject (a) and predicate (b) are disjunct: (a | b)

E.g. "No human (a) is a donkey (b)"

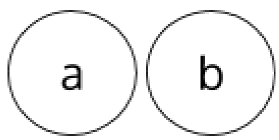

Figure 2: the extension of a *is disjoint with the extension of* b.

I: Particular affirmative: overlap between extension of subject and predicate: (a ◡ b)

E.g. "Someone is running"

O: Particular negative: subject's extension is *not* included in predicate's: (a →/ b)

E.g. "Someone isn't running"

ii. what are modals about?

In modals, the reference of the subject is extended (*ampliatur*) to *possibilia*:

[text 4] A divided [modal] proposition about what is possible [*de possibili*] has its subject

[4] And thus negative propositions about non-existent things come out true: "haec est vera: 'Chimaera non est chimaera'." (*QAM* VI.8, fol.38b).

term ampliated by the modal copula that follows it, so that it stands not only for those things that exist, but also for those that could exist, but don't.[5]

iii. two axioms

There are two axioms that Buridan bases on the foregoing semantics that merit special attention. They are his third and fourth, respectively. In what follows, * and † modify terms: a* ampliates a to what is necessarily-a, a† to what is possibly-a.

axiom III: from necessity to actuality

No assertoric proposition entails one *de necessario*, and neither does a proposition *de necessario* entail an assertoric.

To this there is one exception: universal negatives *de necessario* entail universal negative assertorics.[6]

Exception:

(i) b† | a† ⇒ b | a i.e. ☐E ⇒ E[7]

Rule:

(ii) b† → a* ⇏ b → a i.e. ☐A ⇏ A

(iii) b† ◡ a* ⇏ b ◡ a ☐I ⇏ I

(iv) b† →/ a† ⇏ b →/ a ☐O ⇏ O

1. The reason for (ii)-(iiv) is ampliation, which Buridan captures with disjunctive subjects. For instance, Buridan reads,
   "Every whale is necessarily a mammal"
   as "Everything *that is or can be a whale* is necessarily a mammal"
   which clearly does not entail
   "A whale is (actually) a mammal".

2. The reason for the exception (i) is clear: if b† and a† are disjoint, and b and a are subsets of b† and a†, then b and a are disjoint as well.

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[5] "Propositio divisa de possibili habet subiectum ampliatum per modum sequentem ipsum ad supponendum non solum pro his quae sunt sed etiam pro his quae possunt esse quamvis non sunt." (*Tractatus de Consequentiis* (*TC*) II.4, ll.1-7).

[6] "Tertia conclusio est: ad nullam propositionem de necessario sequi aliquam de inesse vel econverso, praeter quod ad universalem negativam de necessario sequitur universalis negativa de inesse" (*TC* II.6.3 ll.106-8).

[7] I have included these translations because they are somewhat easier to read off. *Caveat autem lector*: they are ambiguous: they could be read as *de re* modals, or as *de dicto*. But here we are concerned only with *de re*.

axiom IV: from actuality to possibility

No proposition *de possibili* entails an assertoric, but an affirmative assertoric entails a corresponding particular *de possibili*.[8]

At present, we're just concerned with the exception:

Exception:

(i) $b \rightarrow a \Rightarrow b\dagger \smile a\dagger$ i.e. $A \Rightarrow \Diamond I$

(ii) $b \smile a \Rightarrow b\dagger \smile a\dagger$ $I \Rightarrow \Diamond I$

II. conditions on frames

In modern modal logic, the modal adverb *necessarily* is interpreted as a quantifier across possible worlds: thus $\Box\varphi$ is interpreted as 'for all worlds, φ'. Following the seminal work of Saul Kripke, in modern modal logic we discuss modal logics in terms of the *frames* that generate their characteristic axioms. A frame $= \langle W, R \rangle$, where W is a set of worlds, and R is a binary relation on W, sometimes called the the *accessibility relation*.

All normal modern modal logics can be modelled on frames. We might wonder, can Buridan's? The answer is *no*.

III. framed-up beyond all recognition

Many commentators have undertaken to provide a possible-worlds semantics for Buridan. But all such undertakings are doomed. In what follows, I'll show why.

i. the reflexivity condition

Take a Kripke frame, $\langle W, R \rangle$. In what follows, we are just concerned with an R that is reflexive, such that for every $w \in W$, wRw. We might express this by saying that every world in a reflexive frame 'sees' itself:

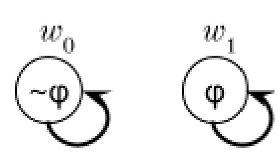

Figure 3: a reflexive model with two worlds.
These worlds can 'see' themselves: at w_0, φ is false; at w_1, it is true.

[8] "Quarta conclusio est: ad nullam propositionem de possibili sequi aliquam de inesse vel econtra, praeter quod ad omnem propositionem affirmativam de inesse sequitur particularis affirmativa de possibili" (*TC* II.6.4 ll.140-2).

To say that φ is necessary is to say that, in all *accessible* possible worlds, φ is true. Suppose for instance that w_1 can only see itself. So at w_1, $\Box\varphi$. (But notice that at w_0, $\Box{\sim}\varphi$. If we opened up w_0, and allowed $w_0 R w_1$, $\Diamond\varphi$ would hold at w_0).

ii. why a possible-worlds semantics for Buridan isn't possible

Let's try to model Buridan's foregoing axioms (III and IV in *TC* I.8). We can start by making our accessibility relation non-reflexive, so that for no world w, wRw. That validates the third axiom: just because something is true in all accessible worlds, doesn't mean it's true in the actual one. To clarify, here's a way our worlds might look:

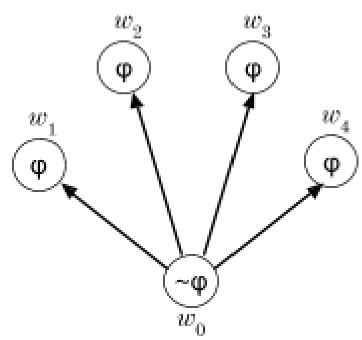

Figure 4: it doesn't matter that φ is false at w_0, since w_0 can't 'see' itself. And in every world w_0 can *see, φ is true; hence at w_0, $\Box\varphi$ is true.*

So we keep axiom III, though we lose the exception (b† | a† ⇒ b | a). And it gets worse: we lose axiom IV, too. Or, rather, we lose its substance: if (b → a) and (b ◡ a) come out false, then each entails anything. So (b → a) entails, *inter alia*, its contrary, b | a!

Reflexive frames or no, we lose one of Buridan's axioms. So a possible worlds semantics just can't do the job here: a terminist modal logic with a possible-objects semantics is able to make distinctions a propositional (or for that matter, predicate) modal logic with a possible-worlds semantics just cannot.

IV. coda: what are these *possibilia*, anyway?

In brief: they're things that causes exist to produce. What sort of causes? The sort Buridan lists in the *Summulae de Demonstrationibus* (6.3), as well as in the *Quaestiones in Analytica Priora* (I.25).[9] Here is a simplified version that combines both:

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[9] "Est enim [I] primus gradus necessitatis quia per nullam potentiam possibile est propositionem falsificari, stante significatione, vel aliter se habere quam significat. [II] Alius gradus est quia impossibile est eam falsificari vel aliter se habere per potentias naturales, licet sit possibile supernaturaliter vel miraculose, ut 'caelum movetur', 'caelum est sphaericum', 'mundus est sphaericus', 'locus est plenus'. [III] Tertius gradus est ex suppositione constantiae subiecti, ut 'lunae eclipsus est propter impositionem terrae inter solem et lunam', 'Socrates est homo',

I. Simple
e.g. "God exists"

II. *De quando* and falsifiable only by divine power
e.g. "The heavens move"
"The cosmos is a sphere"
"There are no vacuums"

III. *De quando* and falsifiable by natural power
e.g. "Socrates is risible"
"Thunder is a sound in the clouds"
"A [lunar] eclipse is an interposition of earth between sun and moon"

NOTATION[10]

$\Rightarrow$ entailment
$\nRightarrow$ non-entailment

* 'necessary' (governing a term: a* is 'what is necessarily a').
† 'possible' (governing a term)
□ 'it is necessary that' (governing a proposition)
◊ 'it is possible that' (governing a proposition)

$\smile$ 'overlaps'
| 'excludes'
$\rightarrow$ 'is included in'
$\rightarrow$/ 'is not included in'

A a universal affirmative proposition
E a universal negative proposition
I a particular affirmative proposition
O a particular negative proposition

a, b terms
φ, ψ propositions

'Socrates est risibilis'. Hae enim dicuntur necessariae sic quia necesse est quandocumque est Socrates, ipsum esse hominem risibilem, et necesse est quandocumque est eclipsis lunae, ipsam esse propter impositionem terrae, etc." (*Summulae de Demonstrationibus* 6.3).

[10] Here I've adopted (and adapted) Paul Thom's notation from his *Medieval Modal Systems* (2003).