Grazer Philosophische Studien
66 (2003), 199–249.

LOGIC AND ANALYTICITY

Tyler BURGE
University of California at Los Angeles

Summary

The view that logic is true independently of a subject matter is criticized—enlarging on Quine's criticisms and adding further ones. It is then argued apriori that full reflective understanding of logic and deductive reasoning requires substantial commitment to mathematical entities. It is emphasized that the objectively apriori connections between deductive reasoning and commitment to mathematics need not be accepted by or even comprehensible to a given deductive reasoner. The relevant connections emerged only slowly in the history of logic. But they can be recognized retrospectively as implicit in logic and deductive reasoning. The paper concludes with discussion of the relevance of its main argument to Kant's question—how is apriori knowledge of a subject matter possible?

Quine's *Two Dogmas of Empiricism* changed the course of philosophy.[1] The defeat of logical positivism freed philosophy to pursue topics to which it had seemed to be closed. Quine's arguments, albeit primarily ones outside the famous paper, subverted the notion of analyticity that had buttressed the positivist view of mathematics and logic. This notion had functioned to close off mathematics and logic from philosophical reflection, and to sever a main route to rationalism and metaphysics. Quine reopened the route, but declined to develop it. The route invites development—especially its epistemic branch. I first survey Quine's criticisms of analyticity in order to evaluate and celebrate his achieve-

1. An ancestor of this paper was given to commemorate the fiftieth anniversary of "Two Dogmas of Empiricism" at a conference in Berlin, during the fateful part of September 2001. I am grateful to Tony Martin for advice and instruction on several issues in sections IV–VI, and to Calvin Normore for help on the pre-Leibnizean historical issues discussed in Appendix II.

ment. Then I consider the place of logic in knowledge of the world. I will argue that in a complex way logic is apriori associated with substantial ontological commitment.

I.

Three concepts of analyticity found a place in philosophy before Quine mounted his attack on the two dogmas of logical positivism. Quine opposed use of all three. Perhaps for this reason he did not bother to distinguish them. The three concepts are not equivalent. They demand different types of treatment. I begin by discussing them.[2]

I call the first the *containment concept* of analyticity. On this concept, a proposition or sentence is analytic if and only if its predicate is contained in its subject. This is Kant's official characterization of analyticity (Kant 1787, A 6/B 10). Leibniz and Kant thought that the truth of the relevant propositions can be demonstrated by analyzing the subject concept so as to find the predicate concept contained within it—as one of its components.

The second is the *logic-with-definitions concept*. On this concept, a proposition or sentence is analytic if and only if it is a truth of logic or can be derived by rules of logic from truths of logic together with definitions or exchanges of synonyms. Leibniz and Kant regarded this concept of analyticity as equivalent to the first, because they assumed that the business of logic was to analyze concepts.[3] Frege freed the logic-with-definitions concept from this assumption, and made this characterization of analyticity his official one (Frege 1884, section 3).[4] This concept carries no commitments about the nature of definitions or of truths of logic.

The third is the *vacuousness concept*. On this concept, a proposition or sentence is analytic if and only if it is true *solely* in virtue of its conceptual content or meaning: a subject matter plays no role in its

2. I distinguish these conceptions in Burge 1992.

3. It appears that Kant, at least, made an exception of certain basic principles of logic, such as the principle of non-contradiction. He counted these principles neither analytic nor synthetic. He thought that their truth could not be obtained by analysis—they were too basic.

4. For further discussion of these matters, including some discussion of Leibniz and Kant, see Burge 2000.

being true; its truth owes nothing to the world. Kant held that logic, but not mathematics, is analytic in this sense. The positivists attributed analyticity in this sense to both logic and mathematics. The vacuousness concept was the centerpiece of their defense of empiricism and their attack on allowing logic or mathematics to engender metaphysics.

The vacuousness concept is my main interest. I think that Quine's attack on it is substantively sound as well as dialectically successful. The dialectical success had a revolutionary effect on philosophy. Logical positivism had tried to extract philosophy from fruitless grandiosity and make it scientific. An effect of this attempt was to constrict philosophy and turn it from legitimate sources of reflection. This is why I see Quine's criticisms as liberating.[5]

The three concepts of analyticity are not equivalent. The logic-with-definitions concept is not extensionally equivalent to the containment concept because many logical truths do not hinge on containment relations among concepts. Many lack subject-predicate form.

The containment concept is not notionally equivalent to the vacuousness concept because it does not entail that containment truths are vacuous.[6] (By "*notional equivalence*" I understand conceptual identity.) Vacuity also does not entail containment. I believe that the containment and vacuousness concepts are not extensionally equivalent either. There are some containment truths ("*That* logical truth is a truth"); but no truths are analytic in the sense of the vacuousness concept.

The logic-with-definitions concept is not notionally equivalent to the vacuousness concept. Being a logical truth does not entail that a subject matter plays no role in its being true. The logic-with-definitions concept is not extensionally equivalent with the vacuousness concept. There are truths of logic, but no vacuously analytic truths.

Although the existence of logical truths suffices for there to be analytic truths on the logic-with-definitions concept, the motive for thus

---

5. Positivism made major *methodological* contributions—its use of logic and insistence on clarity, its interest in language and science, its building intellectual community. These contributions outweigh what I regard as its methodological mistakes—its aversion to traditional philosophical problems and its obsession with reduction, deflation, and dismissal. I believe that Quine deepened the methodological contributions, but passed on the methodological mistakes.

6. Leibniz held that all truths are analytic truths of containment. He also thought that all or most truths are substantive and are made true by the nature of the world. There is no inconsistency in these beliefs. Cf. Leibniz 1677.

counting logical truths analytic arose from assumptions associated with the containment concept. The idea was that all logical truths are deductively "implicit" in the axioms of logic. Frege knew that the containment concept does not apply to all logical truths, because of its narrow focus on subject-predicate form. He still thought that all logical truths are implicitly "contained" in the axioms, in the sense of being derivable from them (Frege 1879, section 13). This view is untenable for logics substantially stronger than first-order logic, by Gödel's incompleteness theorem. So although counting logical truths analytic under the logic-with-definitions concept is harmless, the historical motive for doing so has eroded. At least, the motive must be qualified to apply essentially only to first-order logic.[7]

As noted, Quine rejects use of all three concepts. He opens "Two Dogmas" by identifying the *first dogma* with belief in a distinction between analytic and synthetic truths. The vacuousness concept is evidently at issue. The first dogma is the view that there is a distinctive set of "… truths which are *analytic*, or grounded in meanings independently of matters of fact …" (Quine 1953, 20). Quine proceeds to give little specific attention to the vacuousness concept.

The discussion of the first dogma centers on objections to attempts to explain the notions of meaning, synonymy, and definition. Quine seems mainly concerned with the role of these notions in the logic-with-definitions concept. Some objections lean on requirements of clarity and scientific purity by restrictive standards. Some provide insight into various types of definition and the ways that some types presuppose empirical beliefs. The objections bring out the difficulty of distinguishing between widely shared background knowledge and synonymy.

None of these objections shows that *meaning* or *synonymy* is a defective notion. I think that Quine's requirements on vindicating the relevant notions are inappropriate. The notions have cognitively worthwhile uses in ordinary discourse and in linguistics.[8] Quine's objections were,

---

7. This is a point pressed by Gödel. See Parsons 1995. Gödel had a looser conception of analysis. He regarded difficult axioms for resolving the question of the Continuum Hypothesis as possibly implicit in the concept of set. Yet Gödel did not seem to think of the concept of set as complex or as "containing" components.

Completeness only approximately demarcates the distinction between classical first-order and stronger extensional logics. For an interesting discussion of systems beyond classical first-order logic that have completeness theorems, see Barwise 1977, especially section 5.

8. I believe that Putnam was right to defend a limited use of the notion of synonymy in

however, dialectically effective; for his requirements on vindication were shared by his opponents.

In his attack on the first dogma, Quine so focuses on notions of definition and synonymy that he says little about the idea that logical truths or containment truths are "analytic". As I have said, he appears to ignore the key vacuousness concept almost entirely. These are expositional weaknesses of "Two Dogmas of Empiricism". Quine has a reason for his strategy, however. He thinks that his attack on the second dogma shows that the notion of meaning is cognitively useless. He regards this result as undermining the point of *any* concept of analyticity.

The *second dogma* is the "the belief that each meaningful statement is equivalent to some logical construct upon terms which refer to immediate experience" (Quine 1953, 20; cf. 40 f.).

Quine's argument against it, briefly stated, is this: Meaning is if anything confirmation or infirmation. Confirmation and infirmation are holistic; they apply to whole theories, not to statements taken individually. So meaning is if anything something that attaches to whole theories, not statements, much less words, taken individually.[9]

If sound, this argument would seem to undermine the containment and vacuousness concepts, and the synonymy aspect of the logic-with-definitions concept. So all three concepts are threatened by the argument. This is why Quine writes, "… the one dogma clearly supports the other in this way: as long as it is taken to be significant in general to speak of the confirmation and infirmation of a statement, it seems significant to speak also of a limiting kind of statement which is vacuously confirmed, *ipso facto*, come what may; and such a statement is analytic. The two dogmas are, indeed, at root identical" (Quine 1953, 41).

If it were sound, the argument that centers on confirmation would not undermine counting logical truths analytic, on the logic-with-definitions concept. It would simply eliminate further truths through exchange of synonyms. To maintain that logical truths are analytic under this concept, one need only distinguish logical truths from other truths in some way or other.

---

understanding language. Cf. Putnam 1962.

9. Quine later moderates these views, holding that meaning attaches to blocks of sentences in theories (Quine 1990, 13–17).

Quine can accept this view. Counting logical truths analytic in this sense amounts to counting them logical truths. Quine would make two cautionary points. He would urge that what counts as logic carries an element of stipulation. He thinks that the notion of a logical constant is applicable only relative to given languages: it is not a fully general or scientific notion (Quine 1970, 59). I do not accept this position, but I will not discuss it here.

Quine's second cautionary point would rest on expanding the confirmation argument: Meaning is if anything confirmation or infirmation. Confirmation and infirmation apply to theories, not statements taken individually. All theories face the tribunal of experience. So meaning attaches to empirical theories, not to individual statements.

The expansion of the holism argument asserts empiricism about confirmation. It implies that confirmation in mathematics and logic rests on sense experience. Meaning in these disciplines would depend on role in empirical theory. The cautionary point is that logical truths are just as much confirmed by experience and just as much about the world as truths in natural science. So traditional motives for distinguishing analytic and synthetic truths have no basis.[10]

I think Quine's holism about confirmation insightful and his rejection of the second dogma correct. But I think that both the original and the expanded argument against the second dogma are unsound. Quine had a gift for making these arguments exciting. The metaphors, slogans, and observations invoked to recommend them do not, however, make them cogent. Many of the large ideas in Quine's later philosophy—indeterminacy of translation, inscrutability of reference, ontological relativity, empiricism about logic and mathematics, opposition to non-behaviorist linguistics and psychology, naturalism about epistemology—result from extensions of his argument against the second dogma, combined with his strictures on scientifically acceptable notions. These theses are neither intuitively plausible nor impressively supported by argument. They seem to me no better grounded than the grandiose metaphysics of Whitehead or Bradley. They differ in their expositional clarity and in their motive to clear philosophy of all but natural science. They are, I think, no more worthy of belief.

---

10. This general line is not put in just these terms. But it is often present in Quine's writings—for example, in the last sections of Quine 1953, and in the last pages of Quine 1970.

I will discuss the two arguments from confirmation only cursorily. I think that they do not yield good grounds for rejecting use of *any* of the concepts of analyticity. Then I will turn to what interests me more—Quine's success in attacking the vacuousness concept.

Quine offers no argument that meaning is if anything confirmation. Linguistic meaning partly depends on there being inferential connections. But empirical meaning constitutively depends on other things—on causal relations to an environment, for example.[11] These relations hold independently of the individual's ability to conceptualize them, hence independently of the individual's means of confirming his beliefs. Anti-individualism shows how elements of meaning or content are constituted compatibly with wide variation in theory and confirmatory methods (Burge 1979a; 1982; 1986; 1990; and forthcoming; Putnam 1975; 1988). So meaning partly depends, constitutively, on relations to an environment that are largely theory independent.

The main point is that Quine gives no good reason to think that meaning *is*, if anything, confirmation, or that its partial dependence on inferential capacities makes it assignable only to discourses, not to words and sentences. The view is hardly antecedently plausible.

As to the second premise, Quine is right that empirical confirmation tests several claims together. But he provides no reason, from an account of actual scientific practice, to think that it follows that individual claims lack discrete content. He does not discuss confirmation in enough detail to account for the ways experiments target some claims differently from others. The falsity of the first premise (that meaning is, if anything, confirmation) remains the central problem.

The empiricist premise in the expansion of the confirmation argument—that all statements face the tribunal of experience—is again not argued anywhere in depth. The view rides the waves of assertion and metaphor. Logic and mathematics do not treat their axioms or theorems as hostage to natural science.[12] Knowledge of pure mathematics does not seem to depend on the role of mathematics in empirical explanation. Quine provides no grounds to think that the non-empirical reasons given in the pure mathematical sciences are inadequate on their own terms.

---

11. These relations ultimately depend on perception or interlocution, at least in empirical cases.

12. How mathematics is applied in natural science is largely an empirical matter. This is the lesson of the non-Euclidean geometries. It does not follow that principles of pure mathematics are justified empirically.

So both the spare argument and the expanded argument from confirmation against analyticity limp at every step. Quine does, however, advance powerful criticisms of the vacuousness concept of analyticity. I turn now to those criticisms.

II.

The idea that logic is a science of being is an old one. It dominates the history of logic. Aristotle, most medievals, Leibniz, Frege, Russell, and Gödel maintained versions of it. A contrary view emerged early. Kant credits Epicurus with proposing, against Aristotle, that logic merely supplies norms for thinking—a canon for thought, not an organon of knowledge about a subject matter (Kant 1800, 13). Kant took up this line, introducing the vacuousness concept of analyticity and applying it to logic (Kant 1787, A 55/B 79; A 58–9/B 83; Kant 1800, 94). Carnap and other logical positivists owe to Kant their view that logical truths do not depend for being true on a subject matter. They distinguish themselves from Kant by applying this view to mathematics as well as to logic, and by claiming that such truths originate in linguistic convention, pragmatic decision, or the like.

Quine's strongest arguments against use of the vacuousness concept of analyticity do not occur in "Two Dogmas of Empiricism". They occur in "Truth by Convention", "Carnap and Logical Truth", and *Philosophy of Logic*. I divide these into three arguments or argument-types.

The first argument, advanced in "Truth by Convention", attacks the view that the truth of logical truths is to be explained as a product of convention—hence as vacuous. Quine's counter-argument goes: To cover all logical truths, the supposed conventions must be general: there are too many logical truths to provide conventions for them individually. For particular truths to be true by some convention, they must follow from convention by logical inference. Relevant inferences are understood in terms of their role in preserving truth. Moreover, they are themselves associated with counterpart truths (conditionalizations of the inferences). Appeal to convention cannot explain logical truth since it must presuppose logic (Quine 1936, 97 ff.).

This argument showed that any explanation of logical truth presupposes logic. It devastates its intended target—truth by convention. But it does not defeat all versions of the view that logic is analytic under the vacuousness concept. Carnap waived any pretension to explain logical truth in terms of convention (Carnap 1937, 124). He simply postulated that logic is vacuous. He thought that such postulation best serves the interests of science.

Quine's second type of argument holds such postulation to be of no scientific value. In "Carnap and Logical Truth" Quine discusses three kinds of purported support for the view that logical truths are vacuously analytic. One kind notes that a sentence like "Brutus killed Caesar" owes its truth not only to the killing but to use of component words. It is suggested that a logical truth like "Brutus killed Caesar or it is not the case that Brutus killed Caesar" owes its truth not at all to the killing but purely to the meaning of words—here, "or" and "it is not the case that". A second consideration notes that alternative logics are treated as compatible with standard logic. They use familiar logical words with unfamiliar meanings. This point might be taken to show that logical truths owe their truth entirely to the meanings of logical words. A third consideration is that allegedly pre-logical people are best seen as mistranslated. Translation of someone as committed to a simple contradiction is bad translation. The point might again be taken to show that logical truths depend only on the meanings of logical words (Quine 1954, 101 f.).

Quine shows that all three considerations beg the question. Appeal to the obviousness of logical truths equally well accounts for them (Quine 1954, 105 f.). As to the first, one can just as well regard the logical truth as true not in virtue of the killing, but in virtue of more general obvious traits of everything.[13] No appeal to analyticity is needed. As to the second and third considerations, the obviousness of logical principles again suffices to account for translation practice. Quine writes, "For there can be no stronger evidence of a change in usage than the repudiation of what had been obvious, and no stronger evidence of bad translation than that it translates earnest affirmations into obvious

13. Quine switches here from truths of the form A or $\neg A$, to the truth of the sentence $(x)(x=x)$. This is perhaps because self-identity is more easily seen as a "trait" of things. Quine does not spell out how his point applies to the example from the propositional calculus that he began with. I think that it is dubious that the truth of the initial logical truth is independent of the killing. The truth *would have been* true even if the killing had not taken place, since the truth is necessary. But it does not follow, nor is it obvious, that the killing plays no role in the truth's being true. Simple reflection on the truth condition suggests that it does play such a role.

falsehoods" (Quine 1954, 106). The vacuousness concept provides no explanatory advantage in accounting for the phenomena.

I think these responses brilliantly insightful. To all appearances they are decisive. No genuine support has been given for using the vacuousness concept. I think that no support is forthcoming. In the absence of a reason to distinguish truths that do not owe their truth to a subject matter from truths that do, the use of this concept of analyticity should be rejected.

Quine's third type of argument against the vacuousness concept is more implied than supplied. It is suggested in *Philosophy of Logic*: "Logical theory … is already world-oriented rather than language-oriented; and the truth predicate makes it so" (Quine 1970, 97). It is implicit in the remarks in "Carnap and Logical Truth" about truth depending on a subject matter, and in the concluding metaphor about the lore of our fathers (including their logic) being grey—black with fact and white with convention (Quine 1954, 105 f.; 125). It is suggested by the argument:

> How, given certain circumstances and a certain true sentence, might we hope to show that the sentence was true by virtue of those circumstances? If we could show that the sentence was logically implied by sentences describing those circumstances, could more be asked? But any sentence logically implies the logical truths. Trivially, then, the logical truths are true by virtue of any circumstances you care to name—language, the world, anything. (Quine 1970, 96).

The implied argument goes: Logical theory invokes the notion of truth. Truth is world-oriented. It entails successful relations of reference to or truth-of a subject matter. Any attempt to separate truth from a subject-matter must produce reasons. In the absence of such reasons, logical truths cannot justifiably be regarded as true independently of relation to a subject-matter.

This third argument is the positive counterpart of the negative second type. The argument indicates the deep relation between logical truth and truth of a subject matter. This relation supports associating truth with correspondence. Correspondence has been taken to require a relation between whole sentences or propositions and entities peculiar to them. It requires no such thing. "Correspondence *theory*" often masks pretension. Correspondence is too vague to explain truth. In a sense, nothing explains truth. But understanding truth requires applications of the notions of reference and truth-of. Indeed, understanding any of these three notions requires the others. Attempts to understand truth as "purely formal" or "solely in virtue of meaning" need not only good grounds. They need a reason to think that they are talking about truth at all.

I believe that this third type of argument is sound. It is in the spirit of many of Quine's remarks. There is a presumption of a role for a subject matter in understanding truth. Appeals to analyticity do not confront this presumption, nor do they provide good reasons for doubting it.

Quine's relation to this third argument is equivocal. I believe that he relies on it and often implicates it. But he resists stating it full voice. In his second group of arguments he writes, "We *can* say that ["Everything is self-identical"] depends for its truth on traits of the language (specifically on the usage of "="), and not on traits of its subject matter; but we can also say, alternatively, that it depends on an obvious trait, viz., self-identity, of its subject matter, viz., everything. The tendency of our present reflections is that there is no difference" (Quine 1954, 106). In the same passage where I find the third argument suggested, he writes, "Is logic a compendium of the broadest traits of reality, or is it just an effect of linguistic convention? … [this question] has proved unsound; or all sound, signifying nothing." (Quine 1970, 96).

In these disclaimers Quine holds that "true in virtue of", "depends for its truth on", and "traits of reality" have no use in an explanatory theory. He regards such phrases as explanatorily empty: "Logic is true by virtue of language only as, vacuously, it is true by virtue of anything and everything" (Quine 1970, 97). After "Carnap and Logical Truth" he claims that reference and truth-of are indeterminate. This view requires separate argument. I do not accept it.

The disclaimers are in any case misleading. They fail to note the asymmetry between the two positions. "True in virtue of everything" is too vague to be explanatory. It is, however, connected to both intuitive and formal semantical notions of reference and being true of. It accords with the remark that truth implies "world orientation". "True solely by virtue of meaning" stands unconnected to any such basis. Quine's disclaimers are misleading also in that they can easily seem to be out of keeping with his own position. Logical truth by his lights is just as much about the world, about physical objects and sets, as is physics and mathematics.

III.

In what follows, I assume that logic is not analytic under the vacuousness concept. Like all truths, logical truths depend for their truth on relations to a subject matter. Dependence can be clarified by elaborating the role of "true of" in specifying connections between predicates and variables of quantification, on one hand, and objects, sets, or properties, on the other. Being true requires that sub-propositional elements like predicates bear relations to a subject matter.

Certain special features of logic have tended to provide fallacious encouragement to the view that logical truths are analytic under the vacuousness concept.

Logic seems to abstract from attributions specific to any entities. It does not represent the natures or distinguishing aspects of objects. It has been thought that any science must do this. Since logic does not, it abstracts from attributions to objects. This view has no force. The traditional idea is that logic presents structures or properties common to all objects. Its nature is *not* to specify distinguishing aspects of entities that it is about. To abstract from distinguishing aspects is not to abstract from all relation to a subject matter. Logic is distinctive in this respect.

A second special feature of logic is that it sets normative laws for thought. It has been held that in view of this feature, it says nothing about entities that thought is about. This claim is without force. Setting normative standards for thought is compatible with representing structural aspects of any subject matter for thought. The norms have traditionally been thought to get their purchase through connection to thought's function of aiming at and preserving truth.

A third feature that has encouraged use of the vacuousness concept is that logical truths remain true under substitutions of non-referring non-logical parts. Substituting "centaur" for all occurrences of a simple predicate in a logical truth yields a logical truth.[14] I wish not to go into this matter here. But the following point projects to wide applicability. "(x)(Centaur(x) $\rightarrow$ Centaur(x))" is true because the quantified conditional is true of everything.[15]

---

14. This is, of course, a variant on the first consideration that Quine criticizes in "Carnap and Logical Truth". Kant gives these arguments for analyticity from these three features of logic.

15. Of course, the cited proposition would remain true of everything whether or not

Let us return to the first feature of logic that I mentioned. Given that logic abstracts from attributions specific to any entities, how do we know that there are entities? Does logic provide this knowledge? Should it provide such knowledge, if it is to be an apriori science of being?

The axioms of first-order logic commit it to the existence of entities. The variables of quantification range over a non-empty domain. Non-denoting terms are idealized away. But the existence of free logics, allowing an empty domain, suggests that these points do not establish metaphysical conclusions.[16] One might see classical first-order logic as helping itself to presumption of an existing world.[17] One might conclude that knowledge of existence does not reside in logic, and that free logic best represents knowledge expressed in first-order logic.[18]

It is not obvious that it is a thinkable possibility that there be nothing. We can model fragments of language that fail to make contact with the world, by assigning them an empty domain. It does not follow that it is thinkable much less possible that there be nothing. It is doubtful, however, that one can arrive at any such conclusion from logical principles alone.

I shall pass over logics, like Frege's and Russell's, whose axioms carry strong specific commitments to objects.[19] Modern logics have

---

centaurs were included. That is necessity, not mere truth. The structure-dependent nature of the necessity is certainly relevant to understanding the logical truth, but it does not show that the world, the actual world, is irrelevant to the truth of the logical truth. Cf. note 13.

The logical truth can be known without knowing whether centaurs exist. That is knowledge, not truth. There are interesting issues here that need development. I believe, however, that knowing such truths does not depend on knowing exactly which conditions in the world make the truth true, or on knowing exactly how the truth condition is fulfilled. Cf. Burge 1974.

I think that the existence of non-referential representations, and the truth of propositions containing them, depends on there being referential representations. This is, I believe, a general principle that underlies anti-individualism. I shall not defend this view here.

16. For an example of such a free logic, see Burge 1974. This logic exhibits, I think, the fundamental world-dependence of all logical truth, even allowing for empty domains.

17. Cf. Quine's dismissive attitude toward the requirement that classical logic requires a non-empty domain, in Quine 1970, 52 f.. This attitude is, I think, plausible as a response to a quick inference from the existential commitments of classical logic.

18. See Appendix I—Logic: First-order? Second-order?

19. Frege grounded his commitments to logical objects partly in analogies that led him to postulate truth values as objects. He grounded these commitments partly in the belief that the notion of the extension of a predicate is a logical notion and that his comprehension principle, which connected extensions with predication, was a logical axiom. The analogies do not force ontological commitment (Burge 1986a). The comprehension principle led to

tended to avoid such axioms. To be sure, some thinkers do conceive of logic as having substantial ontological commitments to objects through its axioms. Some regard class theory or even set theory as logic. I have no animus against these conceptions. I will, however, follow standard conceptions of even second-order logic, on which logical principles are true in universes of one object—or no objects at all.

I believe that the standard conceptions have rationales. One is that denials of existence of objects do not seem to contradict principles of deductive reasoning.[20] Another is a normative argument closely related to the first: Logic concerns deductive inference. The principles of deductive inference apply in putative situations where there are few or no objects. They apply in impossible theories or suppositions in which apparently necessary entities (space, time, numbers) are treated as absent. Logical truths should remain true under conditions that can be deductively reasoned about. So logical truths should remain true under metaphysically impossible conditions, including small or empty domains, particularly insofar as these can be understood as sub-parts of structures that do exist. (It does not follow that the logical truths are made true by such conditions, or that their truth does not depend on actual conditions.)

This rationale accords with the intuitive notion *logical validity*—truth correctly explicable in terms of structure characterized by deductively relevant logical form. There may be other tenable conceptions of logic. Perhaps even under the conception that I pursue, there are other routes to existence (cf. note 21.) I shall not explore such routes here.

Logic's being made true by objects does not depend on yielding knowledge that they exist. It could provide knowledge about entities whose existence is knowable only by other means. Logic provides knowledge that whatever there is is self-identical. It might be left to other disciplines to show that there are entities that this knowledge applies to.

---

contradiction. Russell postulated an axiom of infinity as a logical principle, but simultaneously doubted that it had this status. Most subsequent logicians have found the doubt more congenial than the postulation—at least as regards principles of logic.

20. This intuition is a legacy of Kant's criticism of the ontological argument for the existence of God. The intuition is complicated by logics for demonstratives or indexicals, but I believe that these logics presuppose agency (demonstrations or uses) wherever they involve commitment to referents. Even in these cases, I do not believe that existence of agents or objects is a consequence of logical principles, but rather a presupposition of them.

Logic certainly yields no knowledge of the existence of subject-matter-specific kinds—neutrons, amoebae, thought events, symphonies. We know to apply logical structures to such entities only on non-logical grounds. So logic depends on other disciplines for knowledge of many of its applications to the world. Perhaps it so depends for all its applications.

The picture just sketched is as follows: Logical truths are about the world. Their truth depends on relations to entities. To know logical truths, we need not know the exact entities and structures in the world on which their truth depends. One can know the truths to be true by understanding that any entities or structures will fulfill their truth conditions. Other disciplines supply knowledge of existence. Logical truths are true of whatever there is. The existence of some entities, for example, the numbers, may well be necessary. Logic does not adjudicate that point. Existence is known through other means.

IV.

This picture may be accurate as far as it goes. Yet I am not satisfied with it. The dissatisfaction that I want to develop here arises from reflecting on how logic is actually done.[21] A standard semantics for logic requires a set of entities as domain of discourse, assignments of entities in the domain to the terms, and assignments of subsets of the domain to the predicates. Logic relies on a mathematically stronger and more committal meta-theory to systematize applications of its basic underlying notions—*logical validity* and *logical consequence*.

Such meta-theories are committed to mathematical entities. Although various nominalist proposals for doing without numbers or sets (in

21. One source of dissatisfaction that I will not discuss is this. The laws of logic are partly laws of predication and quantification. Predication and quantification are representational operations. They appear to apply to structural aspects of the world—the relation between objects and properties, the relation between quantities of objects and properties, and so on. These seem to be necessary aspects of the world. Logic seems committed to them regardless of what individuals fill these structures. That is, despite the methodological value of Quine's criterion for ontological commitment, it is not obvious that the ontological commitments of logic go purely through quantification on its variables. It is worth reflecting on whether ontological commitments reside in the logical constants and in predication (independently of just what is predicated). This is a complex and old issue. I think it near the heart of understanding logic as a science of being. I will not pursue this aspect of our topic here.

mathematics generally) have been advanced, none are, I think, at all plausible.[22] Apart from parochial ideology, I think it clear that standard, accepted mathematics is true and is committed to abstract entities. The mathematical commitments of the meta-theory of logic are relevant to whether ontological commitment is necessarily and apriori associated with reflective understanding of logic.

The key intuitive notions in understanding logic are *logical validity* of sentences or propositions and *logical consequence* in argument. These are certain conceptions of logical truth and formal deductive preservation of truth, respectively. Logical validity is truth grounded in or correctly explicable in terms of logical form and logical structure. The relevant truths have logical forms that bear semantical relations to entities in relevant structures "in the world". Correct explication of logically valid truth rests on such relations and structures. The relevant truth and truth-preservation are conceived as hinging partly on logical form; and the logical form is associated with semantical relations to structural features of subject matters.

The intuitive notions associate logical form and logical structure with a conception of maximum generality. Sometimes the intuitive notion *logical validity* is glossed as truth formally explicable in any structure, or truth under all conditions, or in all structures (but cf. notes 25, 30.) Similarly, for truth-preservation. The two intuitive notions presuppose conceptions of what count as *logical* forms and structures, and may vary with different conceptions of logic. The generality intuition conditions and helps guide what is to count as logical form and logical structure.[23]

The intuitive notions *logical validity* and *logical consequence* do not occur in logic, as notions expressed by logical constants do. When I say "within logic", I take logic narrowly, to include the axioms and rules of

22. I also do not take seriously fictionalism about mathematics, or denials that mathematical theorems are true. There are technically informed defenses of such views, but I find this sort of philosophy of mathematics so lacking in perspective on knowledge, truth, mathematics, and reason, as not to be worth tilting with. The philosophical motives for such views seem to me thin in relation to the enormity of the program they are supposed to motivate—assimilation of mathematics to fiction. I think that the assumption that mathematics is true and committed to mathematical entities is reasonable and widely held. I shall take it for granted.

23. Hanson 1997 holds that the intuitive concept of logical consequence is a hybrid of concepts of necessity, apriority, and formal generality. One can, of course, construct a hybrid concept of this sort. But I believe that doing so tends to blur matters more than to clarify them. I accept that logical consequences can be known apriori and that at least classical logi-

inference, not the meta-theory. The intuitive meta-notions help indicate the point of logic. They are needed for both intuitive and systematic reflective understanding of logic. They conceptualize intuitions that drive logic's formalizations.

As I will soon explain, the *intuitive* notions *logical validity* and *logical consequence* must be distinguished from the *theoretical* notions *model-theoretic logical validity* and *model-theoretic logical consequence*. The model-theoretic notions not only are not the same

---

cal consequences are necessities. But the notions of formality and generality have emerged as autonomous and fundamental in understanding deductive argument.

Hanson argues that none of the three elements in his hybrid concept suffices to explicate intuitions about logical consequence. He claims that the "most straightforward" version of a formal account of logical consequence validates arguments that are intuitively invalid. The formal account that he discusses is: "The conclusion of an argument is a logical consequence of the premises just in case no argument with the same logical form has true premises and a false conclusion" (366 f.). He gives two examples intended to show the inadequacy of this account. One turns on being agnostic about the truth of the claim that every number has a successor (368 f.). I believe that this example is not worth discussing.

Hanson's second example is more interesting: $(\exists x)(\exists y)(x \neq y)$, hence $(\exists x)(\exists y)(\exists z)(x \neq y \; \& \; y \neq z \; \& \; x \neq z)$. Hanson holds that the formal account takes the conclusion to be a logical consequence of the premise, for "the premise and conclusion … are both true, and, on the usual way of classifying terms as logical and nonlogical, they contain only logical terms. Thus the premise and conclusion are true under all substitutions for their nonlogical terms … and under all interpretations of their nonlogical terms." But he takes the conclusion not to be an intuitive logical consequence of the premise (368–371).

Everything Hanson says about this case seems true. But the intuitive notion *logical consequence* is more substantial than his rendering of it in his formal account (quoted in this note above). The intuitive notion is preservation of truth *grounded in* and (correctly) *explicable* in terms of logical form and logical structure. That is, the basis or ground for preservation of truth lies in logical form and logical structure. Since Frege believed that arithmetical structure *is* logical structure, he would have regarded the argument as valid—in virtue, of course, of other "logical" principles than those made explicit in the first-order argument. Let us assume with Hanson and most logicians, however, that arithmetic is not logic and that the conclusion is intuitively *not* a logical consequence of the premise. Intuitively, the problem with the argument is that its preservation of truth is not grounded in, or explicable in terms of, logical form and logical structure. There is nothing about the logical structure semantically associated with the conclusion that is intuitively grounded in the logical structure semantically associated with the premise. Arithmetic insures that (necessarily) the conclusion is true if the premise is. Indeed it insures the (necessary) truth of the conclusion. But neither the truth of the conclusion nor the connection between premise and conclusion is explicable from what we are hypothesizing to be *logical* form and structure. Standard model theory uses domain variation to account for this intuitive fact. But the intuition that model theory elaborates is that there is nothing about the *logical* forms in the argument, and the *logical* structures semantically associated with those forms, that grounds—or allows an explication of—preservation of truth in the argument.

as the intuitive notions. They do not replace them in our understanding of logic. (The intuitive notions concern a type of truth. The theoretical notions introduce a notion of *truth-in*. The notion *truth* is epistemically more basic than the notion *truth-in*.) The model-theoretic notions do, I believe, bear certain necessary connections to the intuitive notions. They yield systematic, formal elaboration of them. Whereas the intuitive notions help yield intuitive understanding of the point of logic and of various logical principles and inferences, the model-theoretic notions help provide systematic theoretical understanding of both the intuitive notions and the underlying logic.

The intuitive notions *logical validity* and *logical consequence* emerge from sufficiently mature reflection on the practice of ordinary object-level deductive inference. They are *internal* to this practice in the sense that applying them need not take account of information from outside that practice. They conceptualize the practice's own aims and activity by reflection on the practice alone. The same can be said of their more theoretical, model-theoretic counterparts.

By contrast, theories of the sociological functions of tribal practices introduce concepts that are not only not available to the practitioners. They would not be available to sophisticated reflection on the aims of the practice taken by itself. One would need empirical knowledge of societies and of human psychology to explain sociological functions (Burge 1975). This knowledge might attribute to the practice a point that is at odds with any aim that practitioners could arrive at by reflecting just on the cognitive and representational aspects of the practice.

The intuitive notions *logical validity* and *logical consequence* are not like that. Nor are their model-theoretic counterparts. It is certainly possible for beings to engage in deductive inference who do not understand what they are doing from a meta-perspective. Understanding requires an objectification and a correct meta-viewpoint that many who are competent in inference lack. It is even possible, I think, for higher animals and primitive people to engage in simple deductive inference but lack a *capacity* for meta-understanding.

Nonetheless, the notions *logical validity* and *logical consequence* have emerged as the key intuitive notions for understanding the practice and point of logic. They are not the only such notions. They emerged as pre-eminent only slowly in the history of logic.[24]

24. See Appendix II—A Sketch of the Key Intuitive Notions in the History of Logic.

Notions of proof, knowability from proof, strict implication, logical necessity, and other modal notions figured in the development of logic. When conclusions are logical consequences of their premises, they necessarily follow from the premises. Logically valid truths, at least in classical logics, are necessary truths. But the intuitive notions *logical validity* and *logical consequence* are not themselves modal notions, any more than they are proof-theoretic or epistemic notions. Logical validity is a notion of formal truth grounded in or correctly explicable in terms of logical form and logical structure. Excepting the notion of proof, no other notion has been as fruitful in understanding logic. None has yielded as extensive elaboration and systematization. To understand the aims and functions of deductive inference from inside the practice, one must employ the intuitive notions *logical validity* and *logical consequence*. They are part of any full intuitive reflective understanding of logic and deductive inference.

Applying the intuitive notions *logical validity* and *logical consequence* entails applying the notion of truth. Logical validity (of sentences or propositions) is a type of truth. Logical consequence is a type of truth-preservation. Both notions are meta-logical. Inasmuch as they are conceptions of types of truth or truth-preservation, they are *semantical* notions.[25]

---

25. Qualifications apply to the relation between the model-theoretic notions and truth. Model-theoretically valid formulas in pure first-order logic are schemas, not assertable sentences. Such schemas are not true or false. They are only true under all interpretations or true in every model. They are forms that yield truths only when they are made into assertable sentences by filling the schematic markers with non-logical constants. The schemas are, however, used to account for logical *truth* and deductive *preservation of truth*, not merely truth-in or truth-under. A logical truth like "Anything red and square is square" is both intuitively valid and model-theoretically valid. Model-theory aims at understanding such truths, and associated deductive inferences, by systematically elaborating the intuitive notions in a systematic semantics for logic.

One does well not to overestimate the closeness of relation between *true-in* or *true-under*, on one hand, and *true*, on the other. There are models—such as models with domains of one object—which logical truths are true in, but which are not really possible and thus play no role in making the truths true. Moreover, the intended model of a first-order theory may play no role in proving the consistency and completeness of the theory, even though all model-theoretically valid sentences in a first-order theory are true. Although the logic of a strong first-order set theory is provably complete, the theory may not *have* an intended *model*. Intuitive logical validity entails truth; and model-theoretic logical validity, at least for assertable sentences, *should* entail truth (cf. Appendix I, note 28, and the previous paragraph of this note). But proving model-theoretic logical validity need not depend on the intended model, if there is one—much less the "full reality" of the world. [*continued*, p. 218]

I intend "semantical notion" *broadly*. Narrowly, semantical notions apply to relations between *symbols* and what they refer to or otherwise represent. This standard usage often carries an assumption that symbols are the primary bearers of truth. One might hold instead that truth, logical validity, and logical consequence fundamentally concern non-linguistic propositions or thoughts. I will not rely on either position, but I want to allow for the latter one since I hold it. On this view, semantic meta-theory elaborates representational relations between components of propositions (components of thought contents) and entities. These are analogs of the narrowly semantical relations between symbols and what they refer to or are true of. The relations hold between representational propositional components and subject-matter entities. On my broad usage, these analogs are semantical relations. Similarly, because of their essential association with (analogs of) reference and truth-of, the notions *truth*, *logical consequence*, and *logical validity* count as broadly semantical notions whether or not they apply fundamentally to sentences, propositions, or thought contents.[26]

The key intuitive semantical notions are not identical with their model-theoretic counterparts. In the first place, logical validity is a type of truth; logical consequence is a type of truth preservation. The model-theoretic notions involve a closely related but different notion—*truth in* a model or preservation of *truth in* a model.

Logical validity and *logical consequence*, notions for types of *truth* and *truth*-preservation, remain the fundamental notions. Where model-theoretic logical validity or model-theoretic logical consequence applies directly to sentences or propositions that are true or false, the model-theoretic notions must be understood as entailing truth or truth preservation. Where the model-theoretic notions apply to schemas, their relation to truth is more circuitous, but still necessary.

26. One can treat Fregean propositions or thought contents on an analog of the model-theoretic way sentences are treated. A *proposition* is "model-theoretically" valid if every proposition that results from a logical-structure-preserving replacement of non-logical components, where the replacing components apply to individuals or (possibly null) sets of elements, within any set of elements to which the quantifiers are restricted (the analog of a domain of discourse), is true. The notion of truth, as applied to replacement propositions that are restricted in this way, would then be elaborated in terms of the semantics of the parts of the proposition—in a way parallel to the way the truth of a sentence in a model is elaborated. The truth of the whole proposition would be systematically explicated in terms of the semantical contributions of the parts. Note that the "model-theoretically" valid proposition need not be among the propositions, resulting from the replacements, whose being true makes it "model-theoretically" valid.

Secondly, the intuitive notions are inspecific or open-ended about the mathematics relevant to systematically explicating their notion of truth in terms of structure. The mathematics that has become standard is model theory. But other mathematics, for example, a theory in which domains are not sets but classes, or where numbers or functions take the place of sets, can systematize the intuitive notions—to all appearances equally well.

Thirdly, it is not obvious how to equip the model-theoretic notions so as to apply to truth-bearers containing semantical predicates (like "true") and yet avoid paradox. The model-theoretic notions are normally applied to sentences that do not contain semantical predicates. It is prima facie plausible that the intuitive notion *logical validity* applies to sentences or propositions that do contain semantical predicates.[27]

A fourth point applies if second-order logic is counted as logic. It is not mathematically trivial or inferentially immediate that intuitive consistency implies having a model in second-order logic. It seems intuitively consistent to assume the non-existence of certain large sets, inaccessible cardinals. This assumption would block any implication from the assumption of consistency of second-order Zermelo-Fraenkel set theory with the axiom of choice to the theory's having a model. Belief in inaccessible cardinals is a matter of mathematical postulation and discovery. Since the intuitive notion is not immediately and intuitively equivalent to the model-theoretic notion, since reasoning and theory would be needed to show the notions to be equivalent, and since the reasoning and theory would not merely analyze concepts contained in the intuitive notion, they are different notions.[28] Similarly, it is not

27. For discussion of the paradoxes, see Parsons 1974 and Burge 1979. The issue of how the notions of generality and structure that figure in the intuitive notions *logical validity* and *logical consequence* interact with the notion of truth is far from adequately understood.

28. This line is developed by Kreisel 1967. I am indebted to Tony Martin for improving my understanding of these matters. He gave the example of the putative non-existence of inaccessible cardinals blocking the connection between consistency and having a model. Although Martin accepts Kreisel's claim that the intuitive and model-theoretic notions differ, he holds that there is no known plausible *actual* counterexample to their being mathematically equivalent—at least that considerations regarding the lack of a universal set and the size of the set theoretic universe have yielded no such counterexample. This is because, for example, it is mathematically plausible that the relevant inaccessible cardinals exist and that the relevant reflection principle that yields a model whenever a sentence of second-order ZFC is true in the universe of sets, is true. Maintaining the connection between intuitive logical validity and model-theoretic validity in reflection principles is one motivation for

mathematically trivial or inferentially immediate that truth in all models of a sentence of second-order set theory implies its intuitive logical validity. This point follows almost immediately from the point that it is not mathematically trivial or inferentially immediate that intuitive consistency implies having a model.[29]

I emphasize that it does *not* follow from the fact a) that it is not mathematically trivial or inferentially immediate that truth in all models is equivalent to intuitive logical validity, that b) truth in all models in

---

research on large cardinals.

Boolos 1985, 83–87, also rejects identifying the model-theoretic notion of validity with the intuitive notion of validity, and thinks that there is no actual counter-example to mathematical equivalence, as opposed to notional or conceptual identity. I think that he incorrectly identifies the intuitive notion of validity with his notion of supervalidity, a non-semantical notion. Validity of sentences or propositions is conceptually a type of truth, and is essentially a semantical notion.

As will emerge, my main argument does not depend on whether model-theory or some analog can provide a mathematical *equivalent* for the intuitive notion *logical validity*. Model theory or some analog can be necessary for systematic reflective understanding of the intuitive notion *logical validity* even if an equivalence is not forthcoming. It provides systematic, example-based understanding.

29. Follows almost immediately: 1) Suppose that sentence *A* (in a second-order set theory) is intuitively consistent and that it is not trivial or inferentially immediate that *A* has a model. Then trivially 2) it is not trivial or inferentially immediate that *A*'s lacking a model implies *A*'s intuitive inconsistency. Trivially, 3) If *A* is intuitively inconsistent, then the negation of *A* is intuitively valid. Trivially, 4) *A* lacks a model if and only if the negation of *A* is model-theoretically valid. Suppose 5) *A* lacks a model. Then by 2), 3), 4), and 5), we have trivially 6) It is not trivial or inferentially immediate that if the negation of *A* is model-theoretically valid (true in all models), then the negation of *A* is intuitively valid. Thus the notion *truth in all models* is not the same as the intuitive notion *logical validity*.

My annoying insertions of "trivially" through the argument 1)–6) are meant to insure that the reasoning goes through across the operator "it is not trivial or inferentially immediate that".

Cf. Kreisel 1967 for Kreisel's somewhat different exposition of the same conclusion. The example from Martin (note 28) can, of course, be adapted to bear on validity instead of consistency. Boolos 1985, 83, gives a variant of the same claim: the truth of a statement of second-order set theory does not immediately or obviously follow from its model-theoretic validity. But truth follows immediately and intuitively from intuitive validity.

It must be emphasized that the Martin and Boolos points are relevant to distinguishing the intuitive notion *logical validity* from the notion *model-theoretic logical validity* only if second-order logic counts as logic. If second-order logic is not logic, as I am inclined to believe (cf. Appendix I), then the fourth consideration discussed in the text is irrelevant to whether intuitive *logical validity* is the same concept as *model-theoretic logical validity*. The first three considerations still apply. As will become clear, the main argument that I develop in this section and in sections V–VI does not depend on whether second-order logic counts as logic.

second-order set theory is not theoretically or mathematically equivalent with logical validity. The point is that the notion or concept *truth in all models* is different from the intuitive notion or concept *logical validity*.

The intuitive notion *logical validity applied to first-order theories is* mathematically equivalent to the theoretical notion *truth in all models of first-order theories*. Kreisel shows this as follows: Sentences that are derivable from the axioms and rules of first-order logic are intuitively valid. Intuitive logical validity is truth in all structures. So an intuitively valid sentence is true in all structures that are first-order models (i.e. intuitive validity applied to first-order theories implies first-order model-theoretic validity). By the completeness theorem, all sentences that are true in all models of first-order logic (model-theoretically valid in first-order logic) are derivable from the axioms and rules of first-order logic. So a first-order sentence is intuitively valid if and only if it is true in all models of first-order logic.[30] These restricted applications of intuitive validity and model-theoretic validity are equivalent.

Some modern explications of logical truth or deductive inference do not rely on the intuitive notions *logical validity* and *logical consequence*, or on any analog of model theory. There are characterizations strictly in terms of remaining true under substitution of non-logical constants. Such characterizations are not fundamental insofar as they leave unmentioned that the truth of logical truths is systematically explicable by reference to semantical contributions of sub-propositional compo-

---

30. Cf. Kreisel 1967 (reprint), 90 f. It is important to realize that the first step in this reasoning is not a step from a purely syntactical point about proof to an intuitive notion of logical validity. To understand the intuitive logical validity of the axioms and rules of first-order logic, one must consider the logical constants with their intuitive meaning. Kreisel is aware of this point. He takes Frege's axioms and rules of derivability as his paradigm of the first stage of the argument. For Frege, these axioms were, of course, *not* merely syntactic shapes, and the rules were not simply procedures for manipulating syntactic shapes.

Kreisel's second step—that intuitive logical validity is truth in all structures—appears intended as a conceptual identity or notional equivalence. I doubt that the step has this status, because I think it important to distinguish truth from truth-in (cf. note 25). Intuitive logical validity is a type of *truth*, not truth-in. Kreisel's argument goes through, however, even if the second step is conceived as some sort of strong intuitive equivalence, short of conceptual identity. In fact, the argument goes through if the second step is weakened to hold that intuitive logical validity entails truth in all structures.

I believe that reasoning analogous to Kreisel's applies to Fregean propositions or thoughts as distinguished from sentences (cf. note 26).

nents.[31] By themselves they have not led to the wealth of knowledge that semantical methods have led to. Their correctness is evaluated through semantical methods. This is evident in the completeness theorem for first-order logic. The use of syntactic, substitutional explications is parasitic on the more standard semantical notions—intuitive and model-theoretic.

The relatively syntactic explications are less general than model-theoretic explications. They depend on assumptions about the strength of the language—on its having enough and the right kinds of predicates.[32] Such assumptions are not guided by discernible principle. The relatively syntactic characterizations elaborate an aspect of our understanding of logical truth. But they do not elaborate the dependence of logical validity and logical consequence on the semantical fine-structure of logical truth and of deductive truth preservation. They do not elaborate the way logical truth, like all truth, depends on relations to a subject-matter.

Likewise, for non-semantical explications of logical truth or logical "following-from" in terms of strict implication or modality. Both reflection and the development of logic show that key intuitive notions in understanding logic are *logical validity* and *logical consequence*. They are notions of particular types of *truth* and *truth preservation*. The notion of truth presupposes semantical relations between sub-propositional contents (such as predicates) and a subject matter. So the notions of reference, truth-of, term, predicate, and object are constitutively associated with the types of truth attributed by the intuitive notions *logical validity* and *logical consequence*. The intuitive notions are doubly committed to the *structure* of these semantical relations. They are committed through the notion of truth. They are also com-

---

31. Cf. Quine 1970, 49–51. I disagree with Quine's preference for syntactical characterizations in this passage. His preference is associated with a tendency in his later work to favor relatively deflationary conceptions of truth. In Quine 1950, the section "Validity", Quine alludes to reasons like those I cite in the next paragraph for preferring the model-theoretic characterization. There he makes primary use of the model-theoretic characterization. The substitution idea goes back at least to Abaelard (cf. Appendix II). In most of its history, the idea has been associated with the intuitive notion *validity*, not cut off from sub-propositional semantical structure, as it has been in some twentieth-century uses of it.

32. Of course, even these relatively syntactical explications use the notion of truth. Moreover, as Quine points out (1970, 53–56), the exposition of these explications is committed to arithmetic, or that part of set theory that can model arithmetic. For further discussion of the language-dependence of the syntactical characterizations, see Boolos 1975, 50–53. Tarski made many of these points in his original paper, Tarski 1936.

mitted through their construal of logical truth and deductive inference as hinging on sub-propositional aspects of logical form.

Mathematical elaboration of applications of the intuitive notions *logical validity* and *logical consequence* is required in any systematic understanding of the notions. Mathematical elaboration of their applications for any logic beyond monadic predicate logic requires significant ontological commitments. Some sentences of first-order polyadic predicate logic are true in all finite universes but are not intuitively or model-theoretically valid. Their invalidity can be systematically and structurally explained only by appeal to an infinity of entities. Löwenheim's theorem entails that a sentence of first-order polyadic predicate logic is model-theoretically valid if and only if it is model-theoretically valid in the domain of positive integers. By Kreisel's argument—that model-theoretic validity and intuitive validity are equivalent for first-order logic —, application of the intuitive notions *logical validity* and *logical consequence* to first-order logic requires commitment to infinitely many mathematical entities.

Reflective understanding of logical validity and logical consequence is systematic in the completeness theorem for first-order predicate logic. The theorem is that all model-theoretically valid sentences in first-order logic can be proved in the logic. By Kreisel's argument, the theorem shows the same for all intuitively valid sentences in first-order predicate logic. There is room for variation in the exact mathematical commitments needed to prove the completeness theorem. But any such proof requires commitment to infinitely many mathematical entities.[33]

---

33. One can produce accounts of the semantical structure of first-order logical validity and logical consequence and prove obvious analogs of the standard completeness theorem for *model-theoretic* logical validity without using models. One can employ numbers with characterizations (or definitions) on those numbers, and a sufficiently strong semantical vocabulary—instead of sets. One can employ functions instead of sets, or proper classes instead of sets. One can avoid taking the domain to be a set. There are many mathematical possibilities. The intuitive notions *logical validity* and *logical consequence* employ a notion of unrestricted generality. Prima facie, more powerful mathematical systematizations will be more appropriate than less powerful ones because of the commitment to generality in the intuitive notion. In being more restrictive, the mathematically weaker methods are less good candidates, than standard methods, for conceptually natural systematization of the intuitive notions. Some of the weaker methods are parasitic on the standard model-theoretic method in the sense that no one would have come upon them if the completeness theorem had not been proved in the standard way. As I noted (notes 28, 29), it is a mathematically open question whether standard model-theoretic systematizations can model the generality

Summarizing our argument sketch: Full reflective understanding of logic and of deductive practice must use the intuitive semantical notions, *logical validity* and *logical consequence*. Systematic reflective understanding of the use and import of these notions must be mathematical, and thus committed to entities the mathematics is committed to. Even for first-order predicate logic, that commitment is to an infinity of mathematical entities. So full reflective understanding of logic and of deductive practice requires commitment to an infinity of mathematical entities.

The conclusion of this argument does not imply that first-order logic is committed through its bound variables to the existence of entities. The argument applies to free logics.

I quoted with approval Quine's remark that the truth predicate makes logical theory world-oriented. Systematic elaboration of application of the notion of logical truth must presume connection between logical truth and principles for predication of entities, and for grouping and partitioning entities in quantification. Whether or not the truths themselves are quantificationally committed to entities, explaining these matters systematically requires mathematics, which *is* so committed. The mathematical entities help exemplify the subject-matter structure of logical validity and logical consequence. They constitute a ubiquitous subject matter partly in virtue of which logic is true. Logical truths are true of everything. The relevant mathematical structures are, of course, not everything. But they necessarily inform everything else. This is why,

---

of the intuitive notions as applied to second-order languages. Perhaps other mathematical theories will be needed. Even if the standard systematizations do suffice, there will likely be different mathematical systematizations that are equally adequate, in something like the way that different mathematically equivalent explications are, by Church's Thesis, adequate to the mathematically inspecific notion of effective calculability. As I argued above in this section (the second consideration), the open-endedness of the intuitive notions' conceptions of generality and structure count against identifying the intuitive notions with the model-theoretic notions. I think it reasonable to think that no single mathematical explication is uniquely appropriate as explication or "model" of semantical structural relations involved in intuitive logical validity. I will say more about this in sections V and VI.

Of course, it is also a philosophically open question whether to count second-order logic as logic (cf. Appendix I). The important point is that even if one restricts logic to first-order logic, any systematic semantical elaboration of the intuitive semantical notions *logical validity* and *logical consequence* must be committed to at least an infinity of entities. These entities are clearly mathematical. For Löwenheim's theorem and discussion of minimal resources needed to prove the completeness theorem, see Kleene 1964, 389–398.

in meta-logic, they can be seen as representatives or models of other subject matters of which logic is true.

We know objects and properties that logic is about in various ways. Such knowledge derives from common sense, self-knowledge and reflection, physics and the special empirical sciences, and mathematics. Mathematics inevitably applies in these other domains. I have highlighted an application that is constitutively associated with systematic reflective understanding of logic. The application of logic within its own meta-theory to a ubiquitous mathematical subject-matter models logic's being true of more mundane objects and properties.

V.

Most of the components of the argument sketched in section IV are familiar. The ideas that logic is associated with a semantics and that the semantics is committed to mathematical entities are certainly familiar. Seen in broadest outline, the chain of connections in the argument is also familiar. Thus seen, the argument is easy and obvious.

Doubting its soundness is also easy and obvious. One might hold that logic is one thing and its mathematical meta-theory, quite another. One might think that ontological commitments of the meta-theory are completely independent of commitment to logic. I think, however, that the rational chain connecting the commitments is tighter than these dismissive lines suggest.

It is true, in a sense, that logic is one thing and its meta-theory is another. It is true that the ontological commitments of model theory go beyond the quantificational commitments of first-order logic. It is possible to accept the axioms and inference rules of first-order logic and doubt the ontological commitments of model theory. It is possible to "adopt" a free logic—one without any ontological commitments through its quantifiers or singular expressions. It is possible to regard "$2+2=4$" and all of set theory as false, because of their commitments to abstract objects. It is possible to engage in logical inference without having the intuitive concept *logical consequence*, much less its model-theoretic counterpart. These points do not threaten the argument I sketched. The argument concerns rational connections, not whether thinking requires, as a matter of logic or psychology, recognizing and accepting those connections.

In this section I highlight three claims in the argument I sketched. One is that the intuitive notions *logical validity* and *logical consequence* are necessary for reflective understanding of logic. The second is that they are broadly semantical notions. The third is that systematic reflective understanding of these intuitive notions, and through them systematic reflective understanding of logic and deductive reasoning, requires commitment to mathematics. In this section, I also state in more detail the argument sketched in section IV.

I have only a little more to say about the first claim. I noted that other notions figure in understanding logic. But it is hardly controversial that the intuitive notions *logical validity* and *logical consequence* provide essential insight into logic and deductive inference. It is hardly controversial that epistemic and modal notions have been less fruitful in developing and systematizing logic. Such notions certainly cannot replace the intuitive semantical notions. The availability of other notions—strict implication, provability, necessity, necessary truth, knowability from proof, preservation of truth on substitutions in non-logical positions—that mimic or partially coincide with the intuitive semantical notions should not distract from recognition that reflection on these latter notions yielded the practice, formulation, and understanding of logic as we now know it. The basic theorems about logic, beginning with the completeness theorem, derive from reflection on applications of these notions to deductive inference.

The second claim complements the first. A—I would say, the—central point of logic is to formulate principles that underlie deductive inference or proof and that systematize good deductive inference's preservation of truth by virtue of its formal properties. Such principles include truths that themselves depend on deductive form and structure. Both deductive truth-preservation and logical truth depend on and are to be explicated in terms of formal aspects of semantical fine-structure. Successful reflective understanding of logic requires the notions *logical validity* and *logical consequence* because they conceptualize this explanatory aim.[34]

---

34. Hartry Field seems to me to blur the points in this and the preceding paragraph. He rightly distinguishes intuitive notions from model-theoretic notions, but centers attention on intuitive notions of logical necessity and consistency (cf. Field 1989, 30 ff.). He misleadingly cites Kreisel in support of the view that the key intuitive notions in understanding logic are "neither proof-theoretic nor semantical" (Field 1989, 32). Kreisel holds that the key intuitive notions are not "semantical" only in the sense that they are not the notions of model-theory—a point on which all sides here agree. Kreisel does not hold that they are non-semantical in the

The third claim needs more comment. What is the relation between the intuitive notions *logical validity* and *logical consequence*, on one hand, and their model-theoretic counterparts, *model-theoretic logical validity* and *model-theoretic logical consequence*, on the other?

It is possible to make deductive inferences without having the intuitive notions or their model-theoretic counterparts. It is possible to have an intuitive understanding of logic through the intuitive notions without having the model-theoretic notions. The intuitive judgments, both within logic and about logic, are epistemically prior to model-theoretic elaboration of them (cf. Appendices I and II) . There are surely reciprocal epistemic relations between intuitive judgments about validity and the use of model-theoretic notions. But reflective intuitive judgments are usually determinative. Model-theory attempts to understand intuitive judgments systematically.

---

ordinary narrow (or broad) sense that I have discussed. He explicitly recognizes their semantical natures in the ordinary sense of "semantical". He glosses the intuitive notion *logical validity* as "truth in all structures" (cf. Kreisel 1967, reprint, 90). Field takes the fundamental intuitive notion for understanding logic to be implication, which he construes modally. He regards it a matter of choice and convenience whether to formalize the notion as an object-language operator or as a predicate (Field 1989, 34 ff.; 83 ff.). Implication construed as an operator is not a semantical notion. Implication construed as a predicate is, on Field's construal, a semantical notion. I think that the operator notion cannot serve as basis for full reflective understanding of logic *because* it is not semantical. It does not make explicit the connection between logical principles and truth. It cannot be used to reflect on the systematic ways that logical truth and deductive preservation of truth depend on semantical relations to subject matters. History (cf. Appendix II), logical practice, and reflection all indicate that the intuitive semantical notions *logical validity* and *logical consequence* are among the fundamental notions in understanding logic. No argument is given against this view. Whether "the" semantical notion *implication* is the intuitive notion *logical consequence* depends on whether its notion of truth-preservation is construed in terms of structure as opposed to modality. Intuitive notions of implication are notoriously multiform.

Field chooses the modal operator treatment of implication as the key notion for understanding logic. He takes the aim of logic to be that of finding out about logical necessity and possibility. He holds that model theory, like all mathematics, is only a not-literally-true instrument to aid discoveries of these matters (Field 1989 85 ff.; 112). But logic aims to systematize certain types of form-and-structure-dependent truth and truth-preservation. Citing one of its *other* aims—studying logical necessity—and seeing semantics as merely instrumental in achieving that other aim is analogous to claiming that natural science is concerned with prediction or observational adequacy, and then holding that its descriptive and explanatory claims are mere instruments, with no literal truth, for achieving prediction or observational adequacy. I have not argued against instrumentalism or fictionalism regarding mathematics. My argument concerns apriori connections among types of understanding whose commitments I assume to be sound.

At most, certain model-theoretic notions provide theoretically illuminating equivalences with the intuitive notions. Truth in all first-order models (model-theoretic logical validity in such models) is theoretically equivalent to intuitive logical validity of a first-order theory. It is an open philosophical and mathematical question whether the intuitive notion can be given *general* theoretical equivalences in something like the way the notion of effective calculability was provided with such theoretical equivalences in Church's Thesis (cf. Appendix I and notes 28, 29, and 33).

Modeling structures associated with applications of the intuitive semantical notions is, however, part of reflectively understanding them. Model theory yields understanding of the intuitive notions not by defining them, and not necessarily by yielding a precise *general* theoretical equivalence. It yields understanding by offering a systematic way to think of examples—models—of semantically relevant structures, particularly sub-propositional semantical structures, on which logical truth and deductive preservation of truth hinge. If the semantical paradoxes or the nature of set theory prevent one from doing better than this (by obtaining a general theoretical equivalence for logical validity and for logical consequence), one will still have gained essential systematic reflective understanding of the intuitive notions. As in empirical domains, understanding need not consist in theoretical identifications or reductions.

Over the next six paragraphs I give a fuller version of the argument sketched in section IV—the argument that reflective understanding of logic and deductive reasoning requires commitment to mathematics. I believe that each step in the argument is apriori.

The intuitive notions *logical validity* and *logical consequence* are necessary for intuitive reflective understanding of logic and deductive reasoning. Reflection shows that logic is centrally concerned with certain kinds of truth and truth preservation. These kinds turn partly on logical form and are correctly explicated in terms of very general structures, or entities in such structures. This concern is conceptualized in the notions *logical validity* and *logical consequence*.

Full reflective understanding of the intuitive notions and their applications must include understanding of sentential or propositional forms on which logical validity and logical consequence partly hinge. Logical validity and logical consequence depend, even in elementary deductive reasoning, on the natures of *sub-propositional forms*, including predicational and quantificational forms. So full reflective understanding of the notions *logical validity* and *logical consequence*, and their applications to elementary deductive reasoning, requires understanding how sub-propositional forms contribute to logical validity and logical consequence.

Full reflective understanding must include understanding of the *semantics* of sub-propositional forms. The notions *logical validity* and *logical consequence* are semantical. They are notions for certain types of truth and preservation of truth. Truth is necessarily associated with predicates' being true of entities and with other sub-propositional semantical relations. Truth can be fully understood only by understanding the relation between the truth of a whole sentence or proposition and the semantical contributions of its sub-parts.[35] Moreover, logical validity and logical consequence turn on the semantical contributions of sub-propositional elements. For both reasons, reflective understanding of logical validity and logical consequence must include semantical understanding of sub-propositional components.[36]

Full reflective understanding of the intuitive notions *logical validity* and *logical consequence*, including their applications, must be *systematic*. Any two sentences in a language or any two propositions might appear in a single argument. Applications of the intuitive notions turn on a relatively small number of formal features shared among many sentences or propositions (relatively small number because of the great generality of their application). So principles accounting for logical

35. I am well aware that I am not engaging with conceptions of truth that attempt to detach it from the notions *reference* and *truth-of*—for example, some that try to explain truth purely in terms of the truth-schema or in terms of agreeing with what is said. I regard such conceptions as quite obviously inadequate. There are objections to them which I believe have not been convincingly answered (for discussion of some of these, see Davidson 1990). But the conceptual relations between truth and sub-propositional semantical relations like reference and truth-of are sufficiently obvious that I believe that such deflationary conceptions are not serious candidates for fully understanding truth.

36. I believe that accounts of substitutional quantification that avoid a systematic account of the semantical relations involved in quantification are parasitic on our normal conception of quantification, which is constitutively associated with sub-propositional semantical relations (cf. note 35). I shall not argue the point. It has fairly widespread acceptance and considerable plausibility. Relatively "non-semantical" conceptions of quantification and truth are analogous to the relatively "syntactic" substitutional conceptions of logical truth (cf. notes 31 and 32). They are not fundamental. Invoking such alternatives to avoid ontological implications is in the tradition of invoking the vacuousness concept of analyticity.

validity and logical consequence must apply systematically across propositions, or sentences in a language.

Given the complexity of semantical relations between sub-propositional elements and subject-matter structures that are relevant to logical validity and logical consequence, systematic reflective understanding of these relations requires that the semantics be mathematicized and the structures taken to include mathematical structures. Systematic reflective understanding of the relations involved in relatively elementary logical inferences is impossible without invoking a mathematics rich enough to carry commitment to an infinity of mathematical entities (cf. note 33.) This is a rational matter, not simply a psychological matter. So systematic reflective understanding of elementary applications of notions fundamental to an intuitive understanding of logic is rationally committed to an infinity of mathematical entities. Logic, narrowly understood, does not claim that there are infinitely many entities. Its truths can be explicated in, and remain true in, smaller structures. But systematic reflective understanding of elementary intuitive judgments about logical validity and logical consequence demands such commitment.

Comparable systematic understanding can be achieved in semantical frameworks with different ontological commitments. Commitment to classical model-theory is not necessary. But commitment to a semantics including mathematics and to an infinity of mathematical entities *is* rationally necessary for a systematic reflective understanding of the intuitive notions, of logic, and of relatively elementary deductive reasoning.

VI.

Many traditional accounts construed reflective understanding as analysis of containment relations among concepts. I have argued that the model-theoretic concepts are not the same as the intuitive semantical notions. Are the model-theoretic concepts contained in the intuitive concepts *logical validity* and *logical consequence*? Certainly not. This negative answer has three aspects. One concerns the relation between the intuitive concepts and concepts for sub-propositional semantical structure. A second concerns the relation between the intuitive concepts and systematization of structure. A third concerns the relation between the intuitive concepts and the particular mathematics involved in model-theory. I shall discuss these aspects serially.

A conception of sub-propositional semantical structure is constitutively associated with the concept of truth. Having a concept of truth requires being able to apply it. Being able to apply it requires having a concept of proposition or sentence. Being able to apply the concept of truth to a proposition or sentence as such requires being able to conceptualize predications and to determine how predication affects truth, which in turn requires the concept *true of*. A similar point applies to the concept of singular reference. Reciprocally, having the concept *true of* requires having the concept of truth. The concept of truth is more basic in understanding the point of belief, judgment, and inference. The concepts *true of* and *refers to* are more basic in understanding how the truth and logical validity of propositions (or sentences) and preservation of truth in deductions depend on contributions of sub-propositional elements. I see no strong case that either of the concepts *truth* and *truth-of* contains the other. So although the intuitive concept *logical validity* is a concept of a certain type of truth, the concept *logical validity* does not *contain* concepts of sub-propositional semantical structure.

Although not containment-analytic, the conceptual connections here are deep and firm. One could not intuitively understand the notions *logical validity* or *logical consequence* yet be unable to recognize entailments between truth and predicates' being *true of* objects, or between truth and quantifications-on-predications' being true of some, all, or most objects. Intuitive recognition of logical consequence and logical validity requires recognition of formal aspects of propositions and arguments. So although the intuitive meta-logical notions do not contain concepts of sub-propositional semantical structure, they are constitutively associated with them.

The notions *logical validity* and *logical consequence* are constitutively associated with the notion of truth. The notion of truth is constitutively associated with notions of truth-of and reference. These notions are constitutively applicable to sub-propositional semantical structures. Intuitive recognition of such semantical structures can be implicit or confused. Specifying the forms and semantical structures of relational expressions and stacked quantifiers had to await Frege, two millenia after Aristotle. Specification required reflection on applying intuitive semantical notions in a range of inferences. This was Frege's methodological insight.[37] Still, the intuitive bases for systematizing

37. Cf. especially the introduction of Frege 1984 for a statement of the methodology. See

predicational and quantificational forms—and ways that semantical valuations turn on them—were available to apriori reflection, given the intuitive semantical notions, mastery of the inferences, and mastery of relevant logical categories (*predication*, *quantification*).

I turn to the second part of our answer. I think that an imperative to systematize is associated with the intuitive concepts *logical validity* and *logical consequence*. The obvious applicability of these concepts to any sentences or propositions, and to arguments containing any sentences or propositions, and the obvious fact that any given sentence or proposition shares logically relevant form with others, provide intuitive impetus to understand *systematically* the ways that logical validity and logical consequence hinge on sub-propositional semantical structure. Recognition of the need for system derives from reflection. But the materials for such recognition lie in the intuitive notions together with their ordinary applications.

"Together with" is important. System is not contained in the intuitive notions, but implicit in applying them. The mastery of sub-propositional forms in *using* language or theory is what is systematic. The relation between the intuitive notions and the intuitive mastery of logical form is what contains the materials for a systematic elaboration of semantical structure. That is why I said that an *imperative* toward systematization is conceptually associated with the intuitive semantical concepts. The combination of the intuitive semantical notions and the logical forms that ordinary individuals have intuitively mastered is synthetic, not a matter of containment analysis. But the combination is present in intuitive practice. This point is fully compatible with recognizing Frege

---

Frege 1892 and Frege 1891 for examples of the application of the method. In his conception of logical truth as being entirely general and in his appreciation of the role of sub-propositional semantical structure in all truth, including logical truth, Frege employed an intuitive notion of logical validity. This is so even though he regarded semantical notions as dispensable in a fully formed logic. Frege did not employ the full-blown model-theoretic counterparts of the intuitive notions. Since he regarded the real world (including logical objects) as the only basis for evaluating logical truth, he did not allow domain variation (cf. Appendix II and notes 31, 32). He had a different conception of what constitutes truth explicable in terms of structure than standard modern conceptions of the intuitive concept. He thought that logical axioms carry a commitment to an infinity of objects (cf. note 19). These doctrinal differences are important, but hardly entail that he lacked an intuitive notion of logical validity. His notion of logical truth is closer to modern notions—especially in its structural, non-modal cast, and in its association with a deep conception of sub-propositional semantical structure—than some commentators have allowed.

and Tarski's achievements in formulating this systematization, and the achievements of Skolem, Gödel, Tarski, and others in mathematicizing it.

Now to the third aspect of our answer. The intuitive semantical notions do not contain any specific mathematical systematization of their application. In the first place, the intuitive semantical notions are mathematically inspecific. In the second, no one mathematical semantical systematization seems fundamental. Recognition of any of a number of ways of systematically exemplifying logically relevant structure is what is fundamental to understanding the mathematical aspects of logical validity and logical consequence (cf. note 33). Such understanding is deepened by appreciating relations among different mathematical systematizations.

So the notion of truth in a model is not contained in the intuitive notions *logical validity* or *logical consequence*. The constitutive relations are apriori and necessary, but not "analytic" in any of its senses. Some of these relations are accessible only by acquiring new concepts and discoveries. Whether there are theoretical equivalences for the intuitive notions is an open philosophical and mathematical question. Mathematicization of a semantics for these notions nonetheless indicates constitutive aspects of the notions and of the underlying deductive practice.

I distinguish six levels of understanding in logical practice. Outlining them will summarize my construal of the relation of logic's meta-theory to intuitive logical practice.

There is, first, the understanding involved in minimally competent deductive inference. I think that some non-human animals engage in deductive inference. They have perceptual beliefs, memories, and some simple logical constants. They think according to simple rules of inference. They believe no logical truths, lack a concept of logical validity, and lack a concept of truth. They make valid inferences without understanding what they are doing. Whether this is empirically so about some non-human animals, this level of understanding seems conceptually possible. Understanding at this level involves minimal inferential competence with logical constants.

Second, there is the understanding involved in believing what are in fact logical truths, including general ones. Such understanding includes the previous level, but involves a further capacity for generalization and abstraction. Believing logical truths like "Everything red and round is round" (as distinguished from following the associated inference rule applied to non-general thoughts) requires abstracting from the useful. This second level requires a capacity not only for inference but for being compelled to belief in general logical truths.[38]

Third, there is the understanding involved in the use of schematic generalization. This is an abstraction from any actual assertions or truths. Take the ability to consider and use, apart from any application, "If A is a human and A is male, then A is human", or "If Socrates is such and such and so and so, then he is such and such". One abstracts from any particular name substituting for "A", or from any particular predications of Socrates, and one understands the generality of the schematic usage. Understanding *how* to fill in the schematic elements is distinguishable both from quantification and from having a meta-logical perspective that specifies such fillings-in as such. It is like explicitly considering "that is green" abstracted from any application of "that". One can understand such singular expressions apart from applications to particular individuals or properties by knowing how to use the expressions while explicitly abstracting from specific contextual application. Such minimal schematic abstraction can also be distinguished from an ability to conceptualize the semantical relations of the substituted names and predicates (or applications of demonstratives or indexicals) as such. Whether or not this is a developmentally distinct level, it is notionally distinguishable from the second and fourth levels.[39]

38. I think it possible to infer from "That ball is red and round" to "That ball is round", without being able to believe the logical truth "If that ball is red and round, then that ball is round", much less the logical truth "Everything that is red and round, is round". Certainly the capacities are different capacities. Otherwise the deduction theorem would be a tautology.

39. Modern first-order logic is usually formulated with schemas. It is sometimes claimed that first-order logic is distinct from higher-order logics and from the logics of Frege and Russell in being fundamentally *about* sentences and in including a truth predicate (and meta-logical specifications of substitution operations). This claim is mistaken. Certainly, modern model-theory for first-order logic is from a meta-theoretic perspective that has these features. But the schematic formulations of first-order logic allow a perspective within the logic proper (the schematically formulated axioms and the rules of inference) as distinguished from a meta-perspective on the logic. The logic proper is not about sentences and invokes no truth predicate. A minimal non-meta-theoretic understanding of the logic proper involves *knowing how* to fill in forms like "If something is such and such and so and so, then it is such and such" so as to have a logical truth like "If something is red and square, then it is red". It also involves knowing how to fill in schematically stated inference rules so as to make actual non-schematic inferences. This is the third level of understanding. Such

Fourth, there is the understanding involved in using meta-logical concepts. Distinctive of this level is having some conception of logical truth and of arguments' deductively preserving truth.[40] This level requires distinguishing logical truths from others, and deductive arguments from other types. Logical truths and deductive arguments may be conceived as evident, or as necessary, or as provable given rules for using certain "logical" constants. Early formulations of logic are at the third and fourth levels. I leave open whether there are cases of occupying the third level, or even second, without occupying the fourth. The levels are notionally distinct.

Fifth, there is the understanding involved in having and applying the intuitive semantical notions *logical validity* and *logical consequence*. Having them perhaps requires, and certainly historically involved, distinguishing them from other conceptions of logical truth—such as modal conceptions, conceptions of obviousness or certainty, conceptions of proof. The intuitive notions *logical validity* and *logical consequence* associate the specialness of logical truths not only with deductive form and formal structure, but with a very strong type of generality.

Sixth, there is the understanding associated with systematization of these notions in model-theory or related systematic, mathematical, semantical theories. This level of systematic reflective understanding is the level at which mathematicization of logic is necessary.

The view I have argued is this: Engaging in inference does not require having the higher levels of understanding. But understanding is rationally impelled from lower levels to the sixth level, given sufficient conceptual maturity and sufficient reflection purely on relatively elementary deductive inferences. Such inferences harbor the seeds of sixth-level understanding.

---

understanding need not be semantical or otherwise meta-logical. Intuitive and mathematical meta-logic involve ascent beyond the minimal understanding involved in using even modern first-order logic proper.

It might be questioned whether the third level is more sophisticated than the second. Quantification into singular term position is more committal than schematic use of the name. But it seems to me that schematic use of dummy names or unapplied demonstratives is conceptually more sophisticated than existential generalization on the singular position in that it involves an abstraction from natural assertive uses in favor of uses that schematize assertions. The issue invites further exploration. It is not crucial to present purposes.

40. Perhaps there is a level, between the third and fourth levels, at which the ordinary concept of truth is first applied. Some conceptualization of sub-propositional forms and sub-propositional semantical structure would be necessary at such a level.

To outline how sixth-level understanding can be developed out of deductive inference: Making deductive inferences requires having beliefs and conversely. Belief aims at truth; deductive inference aims at truth preservation.[41] Deductive inference is essentially associated with logical truth, inasmuch as a conditional between premises and conclusion of a good deductive inference is a logical truth. Logical truth and deductive inference hinge on sub-propositional form. (I believe that propositional calculus is an abstraction from predicate logic.) Understanding both the nature of truth and the specific nature of logical truth and deductive inference requires understanding sub-propositional semantical structure. Logic has maximally general application. Understanding this generality in combination with the points about semantical structure just made yields intuitive notions *logical validity* and *logical consequence*. The relevance of logic to all permutations of sentences or propositions in inference requires that full understanding of the applications of these notions be systematic. The aim at truth, the form-dependence of deductive inference and logical truth, the connection between all types of truth and sub-propositional semantical structure, the generality of application of logical truth and deductive inference, and the systematic character of logical form are all available to reflection, given conceptual resources appropriate to these essential aspects of deductive inference. The actual problems of systematizing the sub-propositional semantical structure of deductive inference, for relatively simple types of deductive inference available even to adolescent children, force mathematicization.

The development of systematic reflective understanding from reflection on deductive inference is throughout apriori. The connection between the first or second level and the sixth is synthetic, under all three concepts of syntheticity. The sixth level conceptualizes and systematizes constitutive, internal aspects of elementary, non-semantical deductive inference. This fact—not facts about containment—indicates that the commitments of systematic reflective understanding bear on the natures of concepts and inferences involved in lower levels of understanding.

The fact that the concepts used at the higher levels need not be available to individuals with lower-level understanding does not show that the higher-level concepts do not help specify apriori the natures of the lower level ones. Attaining systematic reflective understanding

41. I spell out what I mean by "aim" in more detail in section I of Burge forthcoming.

took centuries. This indicates not that the semantical relations plotted in such understanding do not partly constitute the nature of deductive inference. It indicates that attaining the relevant meta-perspective and the conceptual precision needed to fully understand first-level deductive practice, are difficult matters—not only for current logic students, but for humankind through its history.

VII.

Kant thought that logic is analytic under the vacuousness concept and that mathematics and all other sciences are synthetic. He held logic and mathematics to be apriori. Carnap thought that both logic and mathematics are analytic under the vacuousness concept and that only the natural sciences are synthetic. He agreed with Kant that logic and mathematics are apriori. Quine thought that no truths are analytic under the vacuousness concept, and that no knowledge is apriori. Quine was right about analyticity. Kant and Carnap were right about apriority. Quine's empiricism is badly out of keeping with the way that mathematical knowledge is obtained and justified. Since his attack on the vacuousness concept seems to me decisive, I think that there is synthetic apriori knowledge (in the sense of "synthetic" that contrasts with the vacuousness concept). Both logic and mathematics constitute examples.

Quine opens "Carnap and Logical Truth" with flippant reference to Kant's key question:

> Kant's question "how are synthetic judgments a priori possible?" precipitated the *Critique of Pure Reason*. Question and answer notwithstanding, Mill and others persisted in doubting that such judgments were possible at all. At length some of Kant's own clearest purported instances, drawn from arithmetic, were sweepingly disqualified (or so it seemed …) by Frege's reduction of arithmetic to logic. (Quine 1954, 100)

Contrary to his rhetoric, Quine points out that Frege's purported reduction of arithmetic to logic did not show that arithmetic is analytic under the vacuousness concept. Quine's criticism of that concept shows that both arithmetic and logic are synthetic. How is apriori knowledge of such truths possible? Quine sought to evade Kant's question by maintaining that all knowledge is empirical. As noted, this solution seems unacceptable.

One aspect of Kant's answer to his own question is equally unacceptable. Kant required that synthetic apriori cognition be confined to appearances.[42] He thought that space, time, and number—insofar as they are subject matters for cognition—are mind-dependent structures of our representative powers.[43] He was driven to this view because he thought that one can have apriori cognition only of what one produces. I think that this line is not a serious candidate for the truth.

Kant's question remains. How can one know apriori anything about a subject matter?

The argument of section V. is relevant to giving a partial answer to Kant's question, because of the following point. Logical forms, norms, and structures constitute conditions on the possibility of (propositional) thinking. Thinking is constitutively subject to and informed by logical forms, norms, and structures.[44] Reflection on logic uncovers necessary conditions for thought—conditions involving semantical structure. So by the argument of section V., commitment to mathematics and mathematical entities, in semantical elaboration of logical validity and logical consequence, is part of explaining conditions on the possibility of thinking.

I will not try a full argument for this view here. A sketch depends mainly on additions to the beginning and end of the argument already given: Conditions on the possibility of deductive reasoning are conditions on the possibility of (propositional) thinking. Reflective understanding of conditions on the possibility of deductive reasoning is possible by reflecting only on such reasoning. Such understanding requires the notions *logical validity* and *logical consequence*. It constitutively includes synthetic apriori knowledge—for example, knowledge that certain reasoning is logically valid and that its validity

42. I have transmuted Quine's "judgment" into "knowledge". I skate over the important point that Kant was less concerned with knowledge than with *Erkenntnis* or cognition. These issues are not important for present purposes.

43. I gloss over Kant's distinction between transcendental idealism and empirical realism. I do not think idealism plausible from any point of view, transcendental or otherwise. Note that Kant believed in non-logical apriori principles in physics. The track record of such claims has not been good. But the difficulty of the subject may account for this.

44. I leave open whether *full* first-order predicate logic, or something equivalently rich, is necessary to *all* thought. Animals or young children may think but use only fragments of first-order predicate logic. Quantificational structure rich enough to require mathematicization in any systematic semantical account of it is necessary in adolescent human reasoning, and in science.

is explicable through semantical structures. Systematization of such understanding is rationally required. Systematic understanding of the semantical structure of relatively elementary forms of deductive reasoning is necessarily mathematical and committed to mathematical entities. Such understanding constitutes synthetic apriori knowledge. Such knowledge is of semantical structures that are conditions on the possibility of relatively elementary deductive reasoning —hence on the possibility of relatively elementary thought. So synthetic apriori knowledge of mathematical entities is possible (though not necessarily available to any given thinker) if relatively elementary thinking is possible. Such elementary thinking is possible. Its possibility can be known apriori by reflexively understanding actual thinking—in *cogito*-like instances of it. So the possibility of synthetic apriori knowledge of mathematics is implicit in conditions on the possibility of relatively elementary propositional thinking. Such knowledge is necessary to explaining conditions on the possibility of such thinking.

This conclusion only partially answers Kant's question. It draws an apriori, necessary connection between the possibility of relatively elementary propositional thought and the possibility of synthetic apriori knowledge of mathematics. It is parallel to the aspect of Kant's answer that claims that synthetic apriori cognition is possible because it is necessary to the possibility of explaining and justifying sense experience.[45]

---

45. Kant fixed on experience because he thought that synthetic apriori cognition is possible *only* through certain complex warranting connections to the structure of experience. I think this view incorrect for cognition in mathematics and in several other cases. Perhaps Kant would not have taken sense experience (and its structure) to be the source of all theoretical warrant if he had not regarded logic as analytic in the vacuousness sense.

The notions of explanation and justification are intrinsic to Kant's enterprise. For him, having cognition (*Erkenntnis*) constitutively involves an ability to explain and justify the cognition. Experience is, on his view, a type of cognition.

The present case (explaining conditions on deductive reasoning) and Kant's case (explaining conditions on experience) seem to me to differ in their bearing on epistemic warrant. It is extremely difficult to understand the sense in which an explanation of conditions which make experience possible might provide a warrant that supplements entitlements to perceptual belief (for example, in the context of scepticism). But I believe that there are relatively robust respects in which such explanations can provide supplementary justification. I think that it is even more difficult to discern a sense in which an explanation of the conditions that make deductive reasoning possible might yield a justification that supplements the justification or entitlement involved in competent deductive reasoning. Any such supplementary justification would inevitably be relatively thin. These are complex matters, best discussed elsewhere.

The conclusion does not replace the aspect of Kant's answer rejected earlier. It does not replace the aspect that claims that synthetic apriori cognition is possible because it applies to appearances (Kant 1787, B xvii–xviii; A 26 ff./B 42 ff.). How to bridge the feared gap between thought and subject matter without causal-experiential relations still needs explanation.

The argument that I sketched suggests an approach to confronting the apparent gap between representation and subject matter that Kant's question insinuates. The argument suggests that connection to a subject matter lies in the conditions that make logical inference possible. The fear of a gap derives from an illegitimately subjectivistic starting point—a conception of thinking that does not inquire into the objective conditions that underlie the possibility of thinking.[46]

The fear of a gap is generated from the question, "How can mere thinking yield warranted cognition in the absence of causal relations to a subject matter?" We can know that more goes into conditions on the possibility of "mere thinking" than the subjectivistic starting point recognizes. I believe that the argument given here can be further developed so as to contribute to understanding how aspects of mentality, those involved in relative elementary deductive reasoning, are constitutively associated with warranted apriori cognition of a mathematical subject matter. I will not try to support this claim here, or confront the many issues that arise for it.

I have also not tried to defend the objective truth of logic or mathematics. I have tried to better understand reflective understanding of what we know.

All rational enterprises presuppose logic. Systematic reflective understanding of deductive inference, of logic, and of logical validity and logical consequence each requires mathematics.[47] Whether or not logic is committed through its axioms to the existence of a subject matter, logic is rationally implicated with a subject matter. This connection is available to systematic apriori reflective understanding of applications of the intuitive notions that conceptualize the point and practice of logic. The connection binds inferential structures and norms

---

But in each case, the reflective account is a type of explanation of possibility—possibility of sense experience and possibility of deductive reasoning.

46. There is further discussion of these matters in Burge 1992a.

47. See Appendix III—Poincaré on the Dependence of Logic on Mathematics.

that are constitutive of what it is to be a mind to a subject matter, that of mathematics, that informs all subject matters. By reflection we can know such a connection apriori. We must be able to know it if we are to reflectively and systematically understand relatively simple deductive inference, hence relatively simple propositional representation as of anything at all.[48]

Appendix I—Logic: First-order? Second-order?

In most of this paper I focus on first-order logic. There is, I think, more to the idea that second-order logic must make existential commitments through quantification on predicate position than to the idea that such commitments are made through quantification on individual variables. Predication prima facie implies something to predicate, even bracketing commitment to individuals (cf. note 21.) This intuition counts against interpreting quantification into predicate position as being committed only to pluralities of individuals (cf. Boolos 1975; 1984; 1985). Such interpretations seem artificial. Pluralities of individuals do not seem plausible candidates for what are predicated.

First-order logic demands first attention. The derivability of its valid principles, absent in second-order logic, gives it a feature traditionally regarded as central to logic.

There is the further problem that validity or invalidity in second-order logic, standardly interpreted, can hinge on difficult and unsettled mathematical questions, such as the truth-value of the continuum hypothesis. My concern about counting second-order logic as logic is not about strong ontological commitment in the meta-theory. As sections IV–VI emphasize, the meta-theory of even first-order logic is committed at least to an infinity of entities—though the meta-theory for second-order logic can be forced into vastly larger commitments. My concern is about the lack of transparency, indeed the opacity, to reason of validity and consequence in second-order logic. Logic has traditionally been thought to codify principles of good deductive inference that can elicit agreement through being open to check by any reasonably mature and competent rational being. Given that validity in second-order logic is not only not derivable but very unevident to reason, and given that consequence in

48. I believe that Kant's question demands further answers. I think that the idea that thought is constitutively dependent on connection to mathematical structures is an element in many of them.

second-order logic is equally unevident, it is open to doubt whether it should count as logic. The usefulness of logic in being an arbiter for good reasoning is compromised if what counts as good is as unevident as the truth-value of the continuum hypothesis. Of course, historically, reason has sometimes been slow to recognize what seems retrospectively more nearly evident. And there are different conceptions of logic. My primary argument does not depend on whether second-order logic is counted logic.

For further discussion of these matters, see Shapiro 1991, Jané 1993, Burgess 1993, and Cutler 1997. The latter three give reasons closely related to those just given that in effect support taking first-order logic to be the paradigm logic. Jané and Burgess give further methodological reasons. Cutler emphasizes the completeness theorem more than I do, though I think this consideration very significant. If logical consequence in second-order logic were as rationally transparent as reasoning in intuitive arithmetic, my concern about counting second-order logic as logic would be considerably lessened even though arithmetic is incomplete.

I believe that Jané's gloss of first-order logic as having no "substantive content" is quite mistaken. He gives no non-question-begging ground for it. In fact, the reason he gives is a variant of the first of the three considerations I criticize in section III. I think his characterization of the uses and import of first- and second-order logic is otherwise illuminating.

It should be remembered that the notion of model-theoretic validity is not an autonomous, self-standing conception of logical truth. It depends on an antecedent conception of a logical constant, hence of logic. The notion of model-theoretic validity was introduced as a systematization of an *intuitive* notion, *logical validity* (see below, sections IV–VI). It is thus open to discussion whether truth in all models of second-order logic is *logical* validity. (Here "second-order logic" is a proper name!) Notions of what counts as logic are thus prior to application of the model-theoretic notion *validity*. Of course, the notion *truth in all models* can be used independently of whether it helps explicate any logical notion. But insofar as it is meant to illuminate intuitive logical validity, it presupposes a conception of logic. Indeed, the intuitive notion *logical validity* is guided by an antecedent conception of logic. As I noted, there are probably various legitimate conceptions of logic. I am following what I take to be a mainstream conception that has some claim to being fundamental. These issues are relevant to understanding points made in section IV and in note 28.

Appendix II—A Sketch of the Key Intuitive Notions in the History of Logic

It is at best unclear whether Aristotle had the intuitive notions *logical validity* and *logical consequence*. He surely did not have the theoretical model-theoretic notions. Aristotle worked with a notion of *following-from* (cf. Lear 1980, chapter 1). Aristotle had a broadly semantical notion of truth, a limited notion of logical form, and *perhaps* a commitment to the generality of logic. But it is unclear whether he combined such commitments with a non-modal conception of semantical structure or structure-in-the-world to explicate logical truth or good deductive inference. It is widely doubted that he made use of a notion of formal consequence. His notion of following-from seems to have been modal. His immediate followers concentrated on notions of syllogistic form and modality.

The intuitive notion *logical validity* appears to be present in Abaelard. He understands logical validity for conditional *propositions* non-modally, in terms of containment in virtue of form (Abaelard, ca. 1115, II.iii, 253; 255 f.; III, i, 283 f.). He distinguishes this from the modal conception of the impossibility of a true antecedent and false consequent (ibid., III.i, 271), and understands containment in terms of structural relations in the world. Note that containment is seen as a basis for *explicating* truth in terms of structure. Whereas Abaelard seems clearly to have the notion *logical validity* for propositions, his conception of good deductive inference bears a more equivocal relation to the intuitive notion *logical consequence*. He uses a broad modal notion of following-from to characterize good deductive inference. Since this notion is more liberal than the containment conception of true conditionals, he in effect denies the deduction theorem. For conditionals require containment for their truth, whereas good deductive inferences require only the impossibility of true premises and a false conclusion. So his primary conception of good inference is not in terms of the intuitive notion *logical consequence*, even though he has the intuitive notion *logical validity* for propositions.

Abaelard explicates his notion of formal or *perfect inference*, however, in terms of logical-form-preserving substitution of non-logical terms (ibid., II, iii, 255), a notion later associated with Bolzano and Quine. Abaelard's notion of "perfect" inference joined with his notion of containment gives him an approximation to the notion *logical consequence*. He takes perfect inference to entail the modal "following from". Thus he claims that uniform substitution preserves "consecution" or "following from". This claim mingles structural and modal conceptions. But since he conceives of containment non-modally, he has the resources for a conception of substitutivity of non-logical terms that preserves containment. Containment implies modality but is not explained in terms of it. Such a conception would be a conception of formal logical consequence. Abaelard claims that perfect, formal inferences contrast with imperfect inferences in that the former do *not* hold in virtue of "the nature of things" (*de natura rerum*). This does not mean that they hold independently of factual matters, but rather that they are not grounded in the natures of existing particulars. They are not *de-re-based*, or particular *loci* of modality. They constrain God as well as things (cf. ibid., II, ii, 201; III, 1, 285; 290–305.)

In the twentieth century the Abaelard-Bolzano conception of uniform substitution of non-logical terms is commonly associated with conceptions of logical truth that abstain from explication of truth and truth-preservation in terms of sub-propositional semantical relations to a subject-matter. These are the "relatively syntactic" conceptions discussed in section IV. For Abaelard, containment at the level of propositional content is made true by an analog of containment among states-of-affairs (ibid., III.1, 286 f.).

Unlike Abaelard, Scotus aligns his conceptions of logical truth and good deductive inference. He appears to have both the notion *logical validity* for propositions and the notion (formal) *logical consequence*. He explains these notions in terms of fundamental natural structures of priority in the world. Modality does not appear to be fundamental in these explications. For discussion of Abaelard and Scotus, see Martin 2000 and Martin forthcoming.

Like Abaelard, Buridan has a substitutional conception of formal consequence that bears semantical relations to structures in the world. Buridan explains formal consequence as the impossibility of the conclusion being false if the antecedents are true, under structure-preserving substitution of categorematic terms (cf. Buridan, ca. 1340, I.6.1). He thus mixes modal and structural considerations. But in one place he explains this necessity in terms of the conclusion "never" being false when the antecedents are true—presumably "never" under any substitutions (ibid., VII.4.5). Thus it may be that he sometimes conceives of formal consequence in terms that are fundamentally structural rather than modal (instead of both). He certainly regards the substitutional conception of formal consequence as having a semantical underpinning.

I believe that Leibniz had the intuitive conceptions. But the role of generality in his view is sufficiently complex that a discussion of the issue here would take too much space (cf. Burge 2000, 22 note 23).

Bolzano has the intuitive concepts *logical validity* and *logical consequence*. Like Abaelard and Buridan, Bolzano takes the substitution conception of logical truth to have a semantical underpinning (cf. Bolzano 1837, II, sections 147 f.). For a discussion of the fundamentally semantical character of Bolzano's conception, see Proust 1989.

None of these figures used anything like domain variation in evaluating validity. Domain variation is not necessary to the intuitive notions *logical validity* or *logical consequence*. In fact, Tarski's original paper on logical consequence does not employ domain variation (see Tarski 1936; Etchemendy 1990; Bays 2001).

It must be emphasized that in the history of logic, the intuitive notions *logical validity* and *logical consequence* are sufficiently open-ended to allow for different construals of logicality, of formality, of structure, and of the relevant notion of generality. The intuitive concepts are compatible with various conceptions (cf. notes 23, 33 and 37).

Appendix III—Poincaré on the Dependence of Logic on Mathematics

Poincaré 1908, chapter IV, advances a generically similar point of view as a criticism of Russell's (and by extension, Frege's) logicism. Poincaré argues that since setting up logic requires inductive characterizations of the syntax and of the proof procedures, and since those characterizations presuppose the notion of number, logicism is circular: logic "presupposes" mathematics. Poincaré concentrates on the proof-theoretic aspects of logic. I concentrate on its semantical aspects—those that bear on its synthetic character.

Poincaré's criticism of logicism is not on target, however. There is no definitional or justificational circularity in Frege's or Russell's logicist theory. They did not see the primary justification of a proof, or the reduction of logic to mathematics, as lying at a meta-level in which the syntax or the existence of a proof is specified. It lay for them in the giving of a proof itself. Similarly, the justification of an axiom or theorem lay in the self-evidence of the axioms and the steps of the proof, not in a meta-logical account of logical validity or of logical consequence involved in the proof. Frege's definitions reflect his conception of justification. I make these points in regard to Frege in Burge 1998, without reference to Poincaré. I think a similar point applies to Russell. For a fuller discussion of Poincaré's point, see Parsons 1965 and Goldfarb 2001. Goldfarb also points out what is wrong with Poincaré's objection to Frege. (Although I agree with him on this point, we differ in our construals of Frege's logic and its relation to standard model-theoretic accounts of logic.) The problems for logicism lie not in Poincaré's point but in the question whether what is needed to derive mathematics is in fact logic and in the question whether the relevant derivations constitute an explanation of the *nature* of mathematics.

In any case, the Frege-Russell view of justification seems to me correct.

The primary justification for belief in logical truths lies in logical competence. This is the understanding that I characterized in the text in section VI as second-level understanding—the kind necessary to believe the truths. (Entitlement to deductive inference lies in first-level understanding.) A meta-theoretic account may add supplementary justification. But primarily it deepens understanding (cf. note 45). Inasmuch as the meta-level types of understanding involve concepts not analytically contained in the object-level logical concepts, the logical constants, there is no conceptual circularity. The object-level justifications are autonomous. Poincaré's insight is that a systematic reflective understanding of logic must invoke mathematics. This understanding is synthetic, not analytic.

REFERENCES

Abaelard, Peter, ca. 1115: *Dialectica*, L.M. De Rijk (Ed.), Assen 1956.

Barwise, Jon 1977: "An Introduction to First-Order Logic" in *Handbook of Mathematical Logic*, J. Barwise (Ed.), Amsterdam.

Bays, Tim 2001: "Tarski On Models", *The Journal of Symbolic Logic* 66, 1701–1726.

Bolzano, Bernhard 1837: *Wissenschaftslehre*, Sulzbach.

Boolos, George 1975: "On Second-Order Logic", in *Logic, Logic, and Logic*, R. Jeffrey (Ed.), Cambridge, Mass. 1998.

Boolos, George 1984: "To Be is to Be a Value of a Variable (or to Be Some Values of Some Variables)" in *Logic, Logic, and Logic*, R. Jeffrey (Ed.), Cambridge, Mass. 1998.

Boolos, George 1985: "Nominalist Platonism" in *Logic, Logic, and Logic*, R. Jeffrey (Ed.), Cambridge, Mass. 1998.

Burge, Tyler 1974: "Truth and Singular Terms," *Noûs* 8, 309–325, reprinted in *Philosophical Applications of Free Logic*, K. Lambert (Ed.), Oxford 1991.

Burge, Tyler 1975, "On Knowledge and Convention," *The Philosophical Review* 84, 249–255.

Burge, Tyler 1979: "Semantical Paradox," *The Journal of Philosophy* 76, 169–198.

Burge, Tyler 1979a: "Individualism and the Mental," *Midwest Studies in Philosophy* 4, 73–121.

Burge, Tyler 1982: "Other Bodies," in *Thought and Object*, A.Woodfield (Ed.), London.

Burge, Tyler 1986: “Intellectual Norms and Foundations of Mind,” *The Journal of Philosophy* 83, 697–720.
Burge, Tyler 1986a: “Frege on Truth,” in *Frege Synthesized*, L. Haaparanta and J. Hintikka, (Eds.), Dordrecht.
Burge, Tyler 1990: “Frege on Sense and Linguistic Meaning,” in *The Analytic Tradition*, D. Bell and N. Cooper (Eds.), London.
Burge, Tyler 1992: “Philosophy of Language and Mind: 1950–1990”, *The Philosophical Review* 100, 3–51.
Burge, Tyler 1992a: “Frege on Knowing the Third Realm”, *Mind* 101, 633–650.
Burge, Tyler 1998: “Frege on Knowing the Foundation,” *Mind* 107, 305–347.
Burge, Tyler 2000: “Frege on Apriority” in *New Essays on the A Priori*, P. Boghossian and C. Peacocke (Eds.), Oxford.
Burge, Tyler forthcoming: “Perceptual Entitlement”, *Philosophy and Phenomenological Research*.
Burgess, John 1993: Review of Shapiro, *Foundations without Foundationalism*, *The Journal of Symbolic Logic* 58, 363–365.
Buridan, Jean, ca. 1340: *Summulae de Dialectica*, G. Klima (Transl.), New Haven 2002.
Carnap, Rudolf 1937: *The Logical Syntax of Language*, London.
Cutler, Darcy 1997: Review of Shapiro, *Foundations without Foundationalism*, *Philosophia Mathematica* 5, 71–91.
Davidson, Donald 1990: “The Structure and Content of Truth”, *The Journal of Philosophy* 87, 279–328.
Etchemendy, John 1990: *The Concept of Logical Consequence*, Cambridge, Mass.
Field, Hartry 1989: *Realism, Mathematics, and Modality*, Oxford.
Frege, Gottlob 1879: *Begriffsschrift*, Halle.
Frege, Gottlob 1884: *Foundations of Arithmetic*, J. L. Austin (Transl.), Evanston, Ill. 1968.
Frege, Gottlob 1891: “Function and Concept”, in *The Philosophical Writings of Gottlob Frege*, P. Geach and M. Black (Transl.), Oxford 1966.
Frege, Gottlob 1892: “On Sense and Reference”, in *The Philosophical Writings of Gottlob Frege*, P. Geach and M. Black (Transl.), Oxford 1966.
Goldfarb, Warren 2001: “Frege’s Conception of Logic” in *Future and Pasts: The Analytic Tradition in Philosophy*, J. Floyd and S. Shieh (Eds.), Oxford.
Hanson, William H. 1997: “The Concept of Logical Consequence”, *The Philosophical Review* 106, 365–409.

Jané, Ignacio 1993: "A Critical Appraisal of Second-Order Logic", *History and Philosophy of Logic* 14, 67–86.

Kant, Immanuel 1787: *Critique of Pure Reason*, first edition 1781; second, 1787.

Kant, Immanuel 1800: *Logik Jäsche*, in *Akademie-Ausgabe*, volume IX, Berlin 1923.

Kleene, Stephen Cole 1964: *Introduction to Metamathematics*, Princeton, fourth reprint.

Kreisel, Georg 1967: "Informal Rigour and Completeness Proofs" in *Problems in the Philosophy of Mathematics*, I. Lakatos (Ed.), Amsterdam; reprinted in part with a postscript in *The Philosophy of Mathematics*, J. Hintikka (Ed.), Oxford 1969. Page numbers cited from the reprint.

Lear, Jonathan 1980: *Aristotle and Logical Theory*, Cambridge, England.

Leibniz, Gottfried 1677: "Dialog on Connections of Things and Words" in Leibniz: *Selections*, Philip P. Wiener (Ed.), New York 1951.

Martin, Christopher J. 2000: *Theories of Inference and Entailment in the Middle Ages*, Dissertation, Princeton University.

Martin, Christopher J. forthcoming: "Abaelard's Logic: The Theory of Entailment" in *Cambridge Companion to Abaelard*.

Parsons, Charles 1965: "Frege's Theory of Number" in *Mathematics and Philosophy*, with postscript, Ithaca 1983.

Parsons, Charles 1974: "The Liar Paradox" in *Mathematics in Philosophy*, with postscript, Ithaca 1983.

Parsons, Charles 1995: "Quine and Gödel on Analyticity", in *On Quine: New Essays*, P. Leonardi and M. Santambrogio (Eds.), Cambridge, England.

Poincaré, Henri 1908: *Science et Methode*, Paris.

Proust, Joelle 1989: *Questions of Form: Logic and the Analytic Proposition from Kant to Carnap*, A. A. Brenner (Transl.), Minneapolis.

Putnam, Hilary 1962: "The Analytic and the Synthetic" in his *Philosophical Papers*, volume 2, Cambridge, England 1985.

Putnam, Hilary 1975: "The Meaning of 'Meaning'", in his *Philosophical Papers*, volume 2 Cambridge, England 1985.

Putnam, Hilary 1988: *Representation and Reality*, Cambridge, Mass.

Quine, W. V. 1936: "Truth By Convention" in his *The Ways of Paradox*, New York 1966.

Quine, W. V. 1950: *Methods of Logic*, New York.

Quine, W. V. 1953: "Two Dogmas of Empiricism", in his *From a Logical Point of View*, New York, 1961; originally published 1953, Cambridge, Mass.

Quine, W. V. 1954: "Carnap and Logical Truth" written 1954; large parts first published, in Italian, 1956; first fully published in English 1960; in his *The*

Ways of Paradox, New York 1966.
Quine, W. V. 1970: *Philosophy of Logic*, Englewood Cliffs, New Jersey.
Quine, W. V. 1990: *Pursuit of Truth*, Cambridge, Mass.
Shapiro, Stuart 1991: *Foundations without Foundationalism: A Case for Second-Order Logic*, Oxford.
Tarski, Alfred 1936: "On the Concept of Logical Consequence" in *Logic, Semantics, Metamathematics*, Indianapolis 1983.