

[NOTE TO READERS, IF THERE ARE ANY OUT THERE!]: This paper is still undergoing revisions. Since the workshop discussion I have added the two appendices, and made some other improvements. But there are still revisions that I envision, in addition to further scrutiny of Appendix I. 1) Comment on the complexities introduced by taking seriously Mates' idea that substitution of expressions with the same sense is ok in unembedded oblique contexts, but fails in embedded contexts. 2) Answer Tony Anderson's complaint E-mail 2/29/04 that I haven't sufficiently explained the sense of "C". 3) Fill out argument for first step in reductio, a la pp. 2-3 of letter to David Kaplan. 4) Defend the three fundamental principles on which the reductios depend as principles of thought, cf. p. 4 of letter to Kaplan. 5) Improve notation—avoid double brackets. 6) Possibly explain Peacocke position more fully, as per suggestion of Terry Parsons.]
[CHECK CHECK'S. NOTE that the published version of 1979 contains printer's errors.]

POSTSCRIPT TO "Frege and the Hierarchy"¹ (3/10/04)

Most philosophers who have thought about the matter take it to be unacceptable to be committed to a hierarchy of senses in accounting for embedded oblique or indirect contexts. Carnap rejected basic Fregean principles to avoid the hierarchy.² Dummett rejected what he claimed was an unimportant Fregean principle to avoid the hierarchy.³ Early Terry Parsons maintained that Frege was not committed to the hierarchy, or at least could have avoided it without affecting the basic structure of his theory.⁴ Davidson held that a language committed to a hierarchy is unlearnable.⁵

Church presented a detailed formalized language committed to the hierarchy. He believed, against Carnap, that substantial theoretical losses are incurred in avoiding it.⁶ I believe that Church was right about this and that his point remains applicable to subsequent alternatives. I believe that the hierarchy is deeply grounded in Frege's standpoint, and that avoiding it requires giving up substantial parts of his theory. I also believe that avoiding it requires giving up principles that are motivated, powerful, and attractive—quite independently of Frege's maintaining them. Finally, I believe that it is not true that a language committed to an infinite hierarchy of senses is unlearnable. I think postulation of the hierarchy tenable and not intrinsically problematic.

I do not claim that there is no way around a sense hierarchy. I do believe that the reasons

that favor postulating it have considerable plausibility. I am inclined to believe that at present there is no superior account of thought, or the attribution of thought-- insofar as that attribution fixes on the nature of thoughts and attitudes. So I believe that the "sense" hierarchy should be taken seriously--not merely as a historical curiosity.

Although I do not accept all of Frege's views about senses, I think that he was in many respects on the right track. I think that his notion of sense was not pointed toward understanding linguistic meaning as a common denominator in a socially shared language. It was pointed at understanding thought expressed by language. Such thought is sometimes shared among language users in a community, sometimes not. Frege was mainly concerned that it be shared in a scientific enterprise. I think that the contents of such thought are not in general eternal and mind-independent, as Frege believed. There are other errors in Frege's view. But I think that words express thought components that are abstract and that are distinct from the ordinary referents of the words or thought components. They are representational, and constitute an epistemic perspective on those referents. I also believe that Frege's appeal to oblique contexts in attributions of propositional attitudes is a part of a correct view. Although I will write here of Fregean senses, I think that the structural points that I make about them are, in the main, applicable to abstract representational contents of thought--what I call concepts.⁷ Substituting "thought content" for "sense" sometimes skirts common misunderstandings of Frege's view.

I will not discuss all these issues. Papers by others written since 1979 seem to me to bring out some of the essential points in complementary ways. I do want to say a little more than I did in "Frege and the Hierarchy" about how to think about the hierarchy in an intuitive manner. I also want to explain why a language committed to it is learnable. Finally, I want to discuss some more recent objections to the hierarchy and a response to my paper that attempts to avoid the hierarchy.

I will assume Frege's view that in ordinary, non-embedded attributions of propositional attitudes, the expressions in the subordinate that-clause denote their customary senses, not their customary denotations. Consider again

(1) Bela believes that Opus 132 is a masterpiece.

Let us assume with Frege that the whole that-clause denotes the customary sense (thought content) of the words "Opus 132 is a masterpiece". The words within the that-clause denote their respective customary senses. These senses are components of the customary sense denoted by the that-clause.

It is evident that in using (1), one's understanding of the customary senses of the words in the that-clause plays a role in picking out those very customary senses. On Fregean principles, the customary sense cannot determine itself, since the customary sense determines a truth-value; and a given sense determines a unique s-denotation.⁸

Frege allows words in natural language to shift their senses and denotations with linguistic context. But he takes senses themselves to determine a unique denotation, if they determine any denotation. He explicitly denies that the sense of an expression within the that-clause—its indirect sense—is the same as its customary sense.⁹

It seems to me that inasmuch as the entities denoted by words in the that-clause are different from those denoted by the same words in customary, direct contexts, it is intuitively evident that the modes of presentation associated with words in that-clauses are different from those associated with those words in direct contexts. Thus "Opus 132" as it occurs in customary contexts denotes a string quartet. In a that-clause it denotes a way of thinking about that string quartet. The sense, or way these different denoted entities are thought about should also differ. Similarly, "is a masterpiece" denotes a property of works of art, or (if you insist) a set of works of

art, in customary contexts. In that-clauses, it denotes a way of thinking about that property. Ways of thinking about the property (modes by which thought is presented with the property) and ways of thinking about a way of thinking about the property are surely different.

One should stop thinking of senses as meanings of words and sentences. Senses are ways of thinking, perspectives on entities presented (or purportedly presented) to the mind. The perspective or way of thinking about denotations is different in the two cases. Given that these ways of thinking are not indexical ways of thinking, and given that the denotations are in the actual world, the difference in denotations argues very strongly for their being presented to the mind by different ways of thinking about them. To talk of a sense here as having multiple denotations (or determinations or s-denotations) is, I think, to lose sight of the fundamental role of sense in type-identifying cognitive perspectives on a subject matter.

I believe that “Frege and the Hierarchy” shows that if certain Fregean principles governing sense and denotation are maintained, then either in the object-language or in the meta-language that explains the truth conditions of the object-language, one must distinguish the indirect sense of an expression (the sense it expresses in an unembedded that-clause) from its customary sense. The main principles are the functional compositionality and functional decomposability of senses, the principle that the sense of a sentence determines its truth value, and the substitutivity of co-denoting expressions (in effect, the principle that the truth-value of a sentence is functionally dependent on the denotations of its constituent parts). I shall return to the details.

There does remain the fact that in using a that-clause one relies on one’s understanding of the customary sense of the words within it in determining the indirect denotation (here, determining the representational content of Bela’s thought). To understand the indirect sense of “Opus 132”, one mainly has the customary sense (one’s customary way of thinking about that

string quartet) to go on. So the senses of the words as they occur in the unembedded that-clause must bear a relatively simple relation to the senses that they express when they occur in an embedded that-clause—the indirect senses.

One can see something similar going on in the way quotation marks are normally used. There is a legitimate but non-ordinary usage of quotation marks, especially common among philosophers of logic, according to which they yield a name of the word-shape that they enclose, where the name abstracts completely from any meaning that the word-shape might have. Normally, however, quotation marks are not used to refer to a mere shape. They are used to refer to a word considered as having a meaning or sense in a particular language. This is why quotations are ordinarily translated.¹⁰ Consider the sense of the quotation expression ““Opus 132 is a masterpiece””. The expression normally denotes—or certainly can denote--the English sentence “Opus 132 is a masterpiece”, understood as having its usual meaning or meanings. The quotation is not intended to denote just any shapes that share the shape of those words. It denotes those words as they occur and have sense in English. The quotation expression ““Opus 132 is a masterpiece”” does not have the same sense as the quoted expression “Opus 132 is a masterpiece”. But one’s mastery of the quotation expression (as denoting words with a certain sense) depends only on mastery of the sense of the quoted word and its shape, and on mastery of the sense of quotation marks. The sense of the quotation expression is determined by the sense of quotation marks together with the sense and form of the word(s) quoted.

That-clauses—at least insofar as they are used to characterize the nature of the propositional attitude--are like ordinary quotation expressions, except that they are used to denote only the sense, or representational content, of words within them. Unlike quotation names, that-clause names do not denote the words per se at all.¹¹ Unlike quotation names, that-clauses enable

one to denote the sense shared by different words—for example, words in different languages. A non-embedded that-clause expresses a new sense that determines the customary sense. This new sense is the indirect sense of the word. This sense determines the customary sense of the words within the that-clause, whereas the sense of the words within the that-clause determine the customary denotations of the words. Normally these customary denotations are not senses at all. In fact, the logical type of a sentence within the that-clause differs from the logical type of the that-clause name. In Frege’s view, sentences have truth-values as denotations. But arguably, sentences do not denote anything at all: they simply have truth-values. An analogous point holds of some expressions within the sentence and the expressions that denote their customary senses: Their denotations may be of different type than the senses that they express. This is further reason to take the indirect senses to be distinct from the customary senses.

There is a function from the indirect sense of each expression to the customary sense of that expression: Sense determines denotation (or s-denotation, or determinatum), as usual. That is old news. The important thing to notice is this: The sense of the that-clause--the first indirect sense of the sentence within it--is dependent on nothing other than the customary sense of the words within the that-clause, together with the sense of the that-clause-forming expression (“that”) that is applied to those words.

We form a canonical name of the customary sense by applying the that-clause-forming expression (“that”) to the sentence that follows. The indirect sense is uniquely fixed by this way of denoting the customary sense (assuming that the words in the that-clause occur obliquely). The sense of the canonical name can be uniquely recovered from the customary sense. There is a function from the denotation of the that-clause name--the denotation of the canonical name that denotes the customary sense--to what we might call the canonical sense that determines that

customary sense. So all the talk by Dummett and others about our not knowing what the indirect sense is seems to me quite wayward and unsupportable. We know what the indirect sense is if we can both use the that-clause forming expression (“that”) and understand the sense of the sentential expression to which it applies. Nothing more is needed.

So far we have arrived at two levels of senses—the customary sense and the indirect sense. Many philosophers have thought that one could stop with one level of sense, or at most these two. My paper shows, I think, that one cannot do so without giving up basic Fregean principles.

Given the way senses of these canonical names work, it seems to me evident that there is no particular problem about learning an infinite hierarchy of canonical senses expressed by such canonical names. The sense of a canonical name that denotes a sense at level n (n greater than or equal to 1) is a sense at level $n+1$.

On the simplest account, there is a function (which we might call “the canonical sense of function”) from the sense at level n to the canonical sense at level $n+1$ that determines, or s-denotes, the sense at level n . [NEED NOTE ABOUT MATES PROBLEM—INDIRECT SENSES OF SYNONYMS CAN DIFFER.] Mastery of the that-clause canonical-name-forming device suffices to enable one to introduce and understand an expression that denotes this function. In a first step toward a formalized language, let this expression be “C”.¹² The function denoted by “C” takes senses as arguments and yields, as values, canonical senses that determine or s-denote the argument sense. When “C” is syntactically applied to a canonical name of a sense s , it yields a canonical name of the canonical sense that determines that sense s .¹³ This functional expression could be roughly glossed “the canonical sense of”, except that “C(s)” must be understood not as a description but as a canonical name, if the expression that “C” syntactically applies to is itself a canonical name.¹⁴ Application of “C” can, of course, be iterated.

I maintain the following Principle for Canonical Names of Senses: The canonical name of a sense can be understood only if the sense that it names is understood.

I believe that the relevant canonical names obey a Stronger principle for Canonical Names of Senses: To think the sense of a canonical name of a sense, one must simultaneously think the lowest-level (ultimately, customary) sense in the downward hierarchy associated with the canonical name. For example, to think the sense of the canonical name “C(C(s))”, where “s” is a canonical name of a customary sense, one must think the customary sense denoted by “s”.

Suppose that a formal language has simple canonical names for customary senses of the finite number of primitive expressions that are not themselves canonical names of senses. (So, we now have double the number of primitive expressions.) Suppose that those simple first-level canonical names obey the two principles just stated. They can be understood and thought only by understanding the customary senses themselves. (Moreover, it is sufficient to understand and think them that one understand the customary senses, understand the first-level names for those senses, and understand the “C” expression.)

Suppose that there are devices for composing these names into canonical names of complex customary senses. Suppose that there are compositional principles for “C” that enable one to form canonical names for complex senses out of canonical names for the components of the complex. Suppose that one can iteratively apply C and the principles for composing canonical names of complex senses to produce canonical names of the senses (simple or complex) at any finite level above the first-level. The first-level already has a finite number of canonical names for the customary non-complex senses that can be combined compositionally. One can master this structure by mastering a finite number of canonical names for customary senses, mastering the principles for combining them (and combining any expressions formed by applying functional

expressions to them) into complex names, and mastering “C”.

Thus there is a downward function from senses to the denotations that they determine and an upward function from canonical senses to canonical senses that determine them. Canonical senses are ways of thinking about senses that are grounded in a grasp of the senses that are thought about. Fregean principles bring out that thinking about senses in certain contexts requires that the perspective on a sense—the way of thinking about it—be distinguished from the sense being thought about, even though in this case the perspective is (largely) fixed by the subject matter, the sense thought about.

The point just made is probably most evident, through the analogy to quotation, for canonical names of canonical senses above the first-level. But an analogous point applies to the senses of first-level canonical names of senses. Such canonical names occur obliquely in unembedded that-clauses, or formalize expressions that occur obliquely in unembedded that-clauses. These first-level canonical names denote customary senses. Normally customary senses are ways of thinking that determine or s-denote ordinary objects and properties (like string quartets and properties of them). The customary sense or way of thinking that is denoted by an expression in a that-clause is presented in a canonical way. The sense is thought about from the perspective expressed by a first-level canonical name of it. These canonical names have different senses from senses of other expressions that denote the same customary senses. It helps here, as elsewhere, to remember that senses are fundamentally cognitive perspectives or cognitive modes of presentation. The canonical sense or way of thinking expressed by the first-level canonical name can be understood only by simultaneously understanding the sense or way of thinking that that expressed sense, or way of thinking, determines. (It determines the customary sense that the first-level canonical name denotes.) Thus to understand the sense of the first-level canonical name

“Opus 132”, as it occurs obliquely in an unembedded that-clause, one must simultaneously understand the denoted customary sense of “Opus 132”. Similarly, to understand the sense of the first-level canonical name that formalizes “Opus 132”, as “Opus 132” occurs obliquely in an unembedded that-clause, one must simultaneously understand the customary sense that the formalizing canonical name denotes.

There is no more formal difficulty about learning a language committed to an infinite hierarchy of senses than there is about learning a language that iterates quotation marks. Perhaps a better analogy is this: There is no more difficulty in learning a language committed to an infinite hierarchy of senses than there is about learning a language involving canonical names for the numbers. The higher levels of canonical senses are determined by the customary sense together with iterations of the that-clause (or “C”) construction.

CHECK. SPELL THIS OUT A BIT MORE. SPEAK TO TONY ANDERSON CLAIM THAT WE HAVEN’T GIVEN THE SENSE OF “C”.

According to Frege, each level of application of a that-clause yields a higher-level canonical name and expresses a higher-level canonical sense.¹⁵ Thus

(2) Igor believes that Bela believes that Opus 132 is a masterpiece
attributes to Igor belief in the thought content that Bela believes that Opus 132 is a masterpiece. The that-clause that includes (1), as it occurs in (2), denotes the customary sense of (1). The customary sense of the expression (1) is composed of the customary sense of “Bela”, the customary sense of “believes”, and the customary sense of the that-clause “that Opus 132 is a masterpiece”. The customary sense of this that-clause is a canonical sense that determines the customary sense of the sentence “Opus 132 is a masterpiece”. So in (2) the that-clause “that Opus 132 is a masterpiece” denotes this level-2 canonical sense or cognitive perspective. It

expresses a level-3 canonical sense that determines this level-2 canonical sense. And so on.

The higher-level senses denote ways of thinking about the lower-level senses. At each level one takes up a new attributional perspective on the level below. These ways of thinking, at different levels of canonical sense, differ, however, entirely in the level of embedding of attribution. Thus the differences are needed by, but are exhausted by the levels of logical attribution. The fundamental “line” of content for each canonical upward route from a customary sense is fixed, except for the levels of attribution, by the customary sense itself.

Let me speak to two intuitive objections to the hierarchy. I believe that these objections are easily met, given the upward and downward functional relations among canonical senses.¹⁶

Suppose with Frege that in (2) we make reference to a certain mode of presentation, partly fixed by a level of logical attribution of the belief content which Igor ascribes to Bela. Thus our attribution attributes to Igor a thought that contains a canonical sense that denotes the customary sense of (1). The customary sense of (1) contains a canonical sense that denotes the customary sense of “Opus 132 is a masterpiece”. Thus Igor’s (presumed) belief contains a canonical sense that denotes the (presumed) content of Bela’s belief. And our attribution to Igor contains a (third-level) canonical sense that determines the (presumed) second-level canonical sense that Igor uses to think about Bela’s (presumed) thought content, customarily expressed by “Opus 132 is a masterpiece”.

One might object that this is an unintuitive description of the attribution in (2). One might object on the ground that for (2) to be true, (a) Igor must be thinking that one of Bela’s beliefs has the content Opus 132 is a masterpiece and (b) in thinking this, Igor is employing the same content in thought as Bela would if he were to think that Opus 132 is a masterpiece. It might be further held (c) that Igor is not thinking of the content in some indirect way as the first content

asserted on such and such a page of a particular, named book.

This objection is ineffectual. Frege would agree with (a). That is in effect just what (2) says. That is common ground. Frege would agree with (c). Canonical names do not denote “indirectly” as definite descriptions do. I believe that I have already developed this point in sufficient detail. Frege would agree with (b) in a sense. Igor could not attribute to Bela a belief through a canonical name of that belief content--or canonical sense that determines the belief content--unless he was also thinking the content. To attribute to Bela the content that he does, he must in doing so also think the content of Bela’s presumed belief--the customary sense of “Opus 132 is a masterpiece”. That is how canonical names of content work, as I have been explaining. Igor’s attribution does not itself denote that content. But what he attributes determines it. Further, the attribution can be made only if Igor thinks the content of Bela’s thought himself. That is how he has a grip on his canonical concept of the content. Igor’s attribution itself names the content. But naming it in this canonical way requires also thinking it. For understanding the canonical name requires understanding the customary sense that it names, as well as understanding the customary sense of the iterable construction for forming new canonical names. In our formalization, the iterable construction is “C”. In natural language it is the second (or higher) iteration of the subordinate “that-clause” constructions. This sense determines the function from senses to the senses that are canonical concepts of them.

In linguistic mode: Igor could not understand a canonical name for Bela’s (presumed) belief content if he did not form the first-level canonical name from an exercised mastery of the expressions that customarily express that content. Here there is a close connection between that-clauses, quotation marks, and canonical sense-names. The advantage of the symbolic canonical names is that their iteration maintains a maximum of extensional contexts. The advantage of the

quotation marks and the that-clauses, as denoters of first-level, customary senses, is that they make it clear that understanding the naming device requires a prior understanding of the named entity. It is a requirement on the symbolic canonical names (at the first-level, hence at higher levels) that they be like that-clauses in this respect.

I think it clear that intuition cannot be expected to adjudicate whether the content of Bela's thought is contained in Igor's attribution or, on the contrary, determined by Igor's attribution, where Igor's way of determining (thinking about) Bela's thought requires also thinking it. Determination (s-denotation) is a theoretical concept answerable to structural as well as intuitive concerns. I believe that the Fregean view is equally in accord with intuition on the point. The Fregean view is strongly supported by the structural theoretical principles discussed in "Frege and the Hierarchy". It must be evaluated in terms of the power and explanatory value of those principles. I conclude that this intuitive objection is ineffectual.

A second intuitive objection goes as follows. From (2) and

(3) It is true that Opus 132 is a masterpiece

one can deduce:

(4) Something that Igor believes that Bela believes is true.

If one collapses the levels down to one, one can infer (4) from (2) and (3) by existential generalization. If one retains the levels, the objection goes, one can account for the deductive validity of the inference only through extremely complicated principles.

This objection is incorrect. By utilizing the downward functional structure of canonical names, once a formalization for sense-composition and canonical names is in place, it is a simple matter to produce a simple formalization of the inference. In fact, all that is needed is existential generalization. Explaining the formal notions and producing this formalization would take up

more room in this postscript than the point deserves. The key idea, however, is that the embedded canonical (that-clause) name of the customary sense remains available to quantification because of the downward functional structure of canonical sense names. Thus in the formalization of (2), Igor’s attribution to Bela is specified with the functional expression: C(that Opus 132 is a masterpiece). One can quantify on the whole functional canonical name or on the argument place within it (or indeed onto the positions of the canonical names within the that-clause, which is composed of canonical names of senses).

To make these points a little more explicit: Let pointy brackets yield canonical names of customary senses. Let the hat indicate appropriately formulated syntactic composition. Let (2) be formalized:

(2a) Believes(I, <Bela>^<Believes>^C(<Opus 132 is a masterpiece>))

Then by existential generalization and exportation:

(EG) $\exists y (y = \langle \text{Opus 132 is a masterpiece} \rangle \ \& \ \text{Believes}(I, \langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} C(y)))$

One can then use an obvious formalization of (3) to get (4). To capture the inference from (2) and (3) to (4), one need only quantify into the place of the canonical name “that Opus 132 is a masterpiece” within the larger canonical name, “C(that Opus 132 is a masterpiece)” at (2a).¹⁷ (For more on existential generalization applied to embedded constructions, see Appendix II.)

I have already explained the cognitive difference between the first-level customary senses and the (indirect) second-level canonical senses that determine them. Between the first and second levels, there are often differences in logical category, which clearly correspond to differences in modes of presentation. (For example, the difference between the sense of a sentence and the sense of a singular term denoting that first-level sense.) Moreover, each level of embedded attribution corresponds to different conceptual perspectives. The argument in “Frege

and the Hierarchy” brings out why these distinctions of levels make a logical difference, not just in the relation between the first and second levels, but at each level of attributional embedding.

I believe that collapsing the hierarchy incurs serious costs in a formal representation of attributions of the representational perspectives that constitute propositional attitudes, and in understanding the relation between sense (or cognitive mode of presentation) and denotation.¹⁸ I want to respond to one attempt to circumvent the argument of “Frege and the Hierarchy”.

Christopher Peacocke tries to answer the line of reasoning that I followed in “Frege and the Hierarchy” that limited the use Method I in the service of dispensing with the sense hierarchy.¹⁹ Peacocke’s view is that there is only one level of sense. He thinks that the sense of expressions in that-clauses is “redeployed”. So the customary senses are not only denoted by obliquely occurring expressions in a that-clause. They are expressed by them as well. This is a version of Method I, one that seeks to avoid not only a hierarchy but even a second level of sense, indirect sense. The only senses are customary, first-level senses. As indicated, I think that when one reflects on what senses are—ways of thinking about purported referents—, this view is one of the least plausible ways of avoiding the hierarchy. It seems to me far less plausible than the view that attempts to stop the hierarchy at two levels of sense. I do not accept either way, however.

My objection in “Frege and the Hierarchy” to taking Method I as a way of avoiding the hierarchy centered on difficulties that arise in giving a theory of truth for Method I.²⁰ Peacocke thinks that my objection rests on an oversight about how to give the truth-conditions for natural language belief contexts using Method I. He notes that although I say that the meta-linguistic semantical account that I gave for Method I does not “give” the senses of the object language, but only describes them, this should not be surprising. For the truth-theoretic axioms that I give (in Section III) do not say what the senses of the expressions of the language are. He then goes on to

give a truth theory that does specify the senses.

This answer rests on a misunderstanding of my objection. I never doubted that one can specify the senses of the object language and carry through a truth theory using those specifications. In fact, under the rubric of “translation” into the meta-language I specifically discuss a truth theory that specifies senses. My claim (in section III of “Frege and the Hierarchy”) was that in an extensional meta-language that specifies senses and translates expressions in the object-language into meta-linguistic expressions with the same senses, the problems I raised for Method II will recur in a meta-language capable of providing a systematic semantics for the truth conditions of sentences of a Method I object- language. The problems will recur if one tries to avoid the hierarchy, unless one gives up basic plausible principles about sense and denotation.

The main relevant assumptions invoked in the original article are of three types. One is that the extensional substitution principles needed in a truth theory are maintained. This is the syntactical counterpart of the principle that a sentence’s truth-value is fixed by the denotations of its parts. The syntactical counterpart is needed to carry out standard proofs within a truth-theory for any ordinary object-language. A second type of principle is that sense determines denotation. For the argument, I only needed an instance of this principle: that sentences with the same sense have the same truth value. The third type is that the translations are compositional and that principles of functional composition and decomposition of senses are maintained in the metalanguage. All of the relevant principles are assumed in Peacocke’s meta-language. Peacocke does not address the claim of section III of the 1979 paper, much less answer it.

I will sketch some arguments that show how the meta-language that Peacocke relies upon leads to the same difficulty that I discussed with respect to Method II languages. The first argument faces a pair of problems. The two problems are shown not to be fundamental by

considering two further arguments, the last of which seems to me to show decisively that my original criticism of the position applies to Peacocke's meta-language. A fully formalized version of the arguments would be more perspicuous, but would also take up more space.²¹

Let " $\langle \dots \rangle$ " be a canonical name of the customary sense of the expression that fills in for the dots. (This is Peacocke's terminology.) So " $\langle \text{Opus 132} \rangle$ " [CHECK—make the "L's subscripts here and in what follows.] denotes the customary sense of "Opus 132" as it occurs in Method I, formalizing object-language L. Let " $\langle \dots \rangle^{\wedge} \langle \dots \rangle^{\wedge} \langle \dots \rangle$ " denote the result of functionally composing the senses named by the bracketed expressions into a canonical name of the grammatically appropriate complex expression.²² For example, " $\langle \text{Opus 132} \rangle^{\wedge} \langle \text{Masterpiece} \rangle$ " canonically names the customary sense of the sentence natural language sentence "Opus 132 is a masterpiece", as those expressions (subscripted by "L") occur in the Method I, formalizing object-language L. Such compositional principles apply to the translations of L's sentences into the meta-language. So for example " $\langle \text{Opus 132} \rangle^{\wedge} \langle \text{Masterpiece} \rangle$ " also names the sense of the meta-language sentence "Masterpiece(Opus 132)", since this meta-language sentence translates the object-language "Opus 132 is a masterpiece". And " $\langle \text{Masterpiece(Opus 132)} \rangle$ " names this same sense, by way of the meta-linguistic expressions that express it.

According to Peacocke's view,

a) The customary sense of "Bela believes that Opus 132 is a masterpiece" =

$\langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} \langle \text{Opus 132 is a masterpiece} \rangle$

a) combines the beginning of a functional decomposition of the sense of (1), "Bela believes that Opus 132 is a masterpiece", with Peacocke's view that in the that-clause of (1), the customary sense of "Opus 132 is a masterpiece" is expressed. That is, what is expressed is the customary

sense, rather than a further indirect sense.²³

Now suppose that the sense of “Opus 132 is a masterpiece” (in the natural language) and the sense of its counterpart in the Method I, formalizing object-language are the same as the sense of the sentence that translates it, “Masterpiece(Opus132)”, into the meta-language. And suppose that the senses of “BelaL” and “BelievesL” are the same as their meta-linguistic counterparts. So translation is normal but tries to capture Peacocke’s view of the object-language L, according to which there is only one level of sense.

Then

$$\text{b) } \langle \text{BelaL} \rangle \wedge \langle \text{BelievesL} \rangle \wedge \langle \text{Opus 132L is a masterpieceL} \rangle = \\ \langle \text{Bela} \rangle \wedge \langle \text{Believes} \rangle \wedge \langle \text{Masterpiece(Opus 132)} \rangle$$

$$\text{c) } \langle \text{Bela} \rangle \wedge \langle \text{Believes} \rangle \wedge \langle \text{Masterpiece(Opus 132)} \rangle = \\ \langle \text{Believes(Bela, Masterpiece(Opus 132))} \rangle$$

c) follows from b) by the functional compositionality of sense. The sense of the expression within the pointed brackets is functionally dependent on the senses expressed by the constituent parts of the expression.

$$\text{d) The customary sense of “Bela believes that Opus 132 is a masterpiece”} = \\ \langle \text{Believes(Bela, Masterpiece(Opus 132))} \rangle$$

d) follows from a)-c) by transitivity of identity.

$$\text{e) “Bela believes that Opus 132 is a masterpiece” is true in L } \longleftrightarrow \\ \text{Believes(Bela, Masterpiece(Opus 132))}$$

e) follows from d) by the principle that sentences with the same sense are materially equivalent. Suppose that “Masterpiece(Opus 132)” is true. Let “S” be any other true sentence in the meta-language. Then

f) “Bela believes that Opus 132 is a masterpiece” is true in L \longleftrightarrow

Believes(Bela, S).

f) follows from d) by the extensionality of substitution principles in the meta-language and substitution of “S” for “Masterpiece(Opus 132)”.

Similarly, since “Opus 132” occurs in extensional position in the meta-language sentence that expresses the sense $\langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} \langle \text{Masterpiece(Opus 132)} \rangle$, singular terms with the same denotation are interchangeable in sentences that express attributions of belief.

f) is a reductio ad absurdum. What does it reduce to absurdity? The principles of the functional compositionality and decomposability of sense and the principle that if the senses of sentences are the same, they have the same truth value, seem very plausible. The extensionality of substitution principles in the meta-language is necessary for giving a systematic theory of truth.

I see two ways of pinning the absurdity of the conclusion on something other than the assumption that the customary sense (not the indirect sense) of a sentence is expressed in its occurrences in object-language that-clauses. Neither of these ways will ultimately suffice.

One way is to complain that the application of the functional compositionality principle to get c) has something wrong with it. One might hold that it should be ungrammatical to attach “Believes” syntactically to a singular term (“Bela”) and a sentence (“Masterpiece(Opus 132)”). This complaint seems *prima facie* strained. We have matched the sense of the object-language’s that-clause with a sentence in the meta-language with the same sense. We get to determine the grammar of the meta-language in such a way as to match our principles of sense and denotation. Moreover, Frege took sentences to be of the same grammatical form (for some purposes) as terms. Surely the issue goes deeper than denying this view.

Still, one might insist that in the object-language, “BelievesL” syntactically applies to a

pair of singular terms (though one of them has the sense of a sentence!), whereas we have made the second open spot of “Believes” in the meta-language a spot for a sentence.²⁴ And one might insist against Frege on not allowing singular terms and sentences to count as members of the same syntactic kind in any sense.²⁵ The “^” notation was introduced as composing senses into a name of an expression of a “grammatically appropriate category”. One could claim that in composing the senses of sentences involving “Believes” in the meta-language, one must produce the sense of a sentence in which a singular expression occurs in the second open position of “Believes”. This singular expression must both denote and express the customary sense of the sentence “Masterpiece(Opus 132)”. Such composition will involve explicit use in the meta-language of singular terms that denote the customary sense of the sentence, but express that same sense.

I will return to this sort of idea and criticize it. Here I want to show that the issue of the grammar of “Believes” in the meta-language cannot be the central matter. For an argument exactly parallel to a)-f) can be applied to the object-language sentence “Byron searched for Ossian”. In direct contexts, “Ossian” and “Tlaloc” lack a denotation. “Byron searched for Ossian” can be true while “Byron searched for Tlaloc” is false. Other singular terms can be used to make the point without using ordinary proper names.

The Fregean treatment holds that a term that follows “searched for” that resists co-extensional substitution denotes its customary sense. Here there is no change from the grammar of direct contexts to the grammar of indirect contexts. There is a singular term in both cases. The analog of the original argument still produces an absurd conclusion (“Bela searched for Ossian” is true in L if and only if Bela searched for T, where “T” is any term co-extensive with “Ossian”, or co-extensive with whatever other term occurs in the position of “Ossian”). This is an absurd consequence that does not at all depend on the ambivalent grammatical position that the

view under discussion involves (and forces) in its explanation of the roles of expressions in ordinary that-clauses. So the grammatical issue cannot be fundamental.

The second way of pinning the absurdity on something other than the collapse of the hierarchy is to reject an assumption behind step b) of the argument—the assumption that “Believes” in the meta-language has the same sense as the object-language “BelievesL”. One can plausibly argue as follows. It turns out that, in the meta-language, “Believes” works so that if a person “Believes” (in scare quotes) one truth, he or she believes them all. So what could show more clearly that the meta-language expression “Believes” does not have the same sense as the object-language “believes” (“BelievesL”)? Similarly, the absurd results in e) and f) simply bring out that the truth-theory rests on bad translation from the object-language into the meta-language.

Now I think that this point is correct. But the question is what is leading to this state of affairs? We used translation of the sentence “Opus 132 is a masterpiece” into “Masterpiece(Opus 132)”, which seems entirely correct. Those sentences in the respective languages (object-language and meta-language) express the same sense. And we applied the composition principle with the resources that we had. We need to consider whether good translation is possible, consistent with the principles that we are assuming. This was the dialectical situation that I envisioned in “Frege and the Hierarchy”. The argument developed so far is a variant on the one given in the original 1979 article. I claimed that if one produces a translation, one will either find the hierarchy in the meta-language, or run afoul of the relevant Fregean (and otherwise plausible) principles. Let us investigate this matter further. I believe that the problem in translation is just a symptom of the more fundamental disease.

What we need in the meta-language is a term that denotes the customary sense of “Masterpiece(Opus 132)”, but does not introduce a further indirect sense. It expresses the sense

that it denotes. Perhaps this is the analog in the meta-language of the way the object-language's that-clauses are supposed to work. This is an idea that we came upon in discussing the first possible difficulty with a)-f), the grammatical difficulty. So let us follow it out here.

We continue to take “<Masterpiece(Opus 132)>” to be a singular term in the meta-language that denotes the customary sense of the sentence “Masterpiece(Opus 132)”. But this singular term has no further sense: its sense is the same as the sense of the sentence. This is in itself an absurdity. The sense of the sentence cannot be the same as the sense of a singular term that is about a sense or a way of thinking. The sense of the sentence is a way of thinking about a string quartet and about its being a masterpiece. The sense determines a string quartet and its being a masterpiece. The sense of the relevant singular term, “<Masterpiece(Opus 132)>”, is a way of thinking about these ways of thinking.

One needs to bear firmly in mind what senses are. They are perspectival ways in which a referent (if any) is presented to the mind. The problem is not merely that the one-level-of-sense view gives up the principle that a sense determines a referent or denotation. It is that by the nature of sense, a (non-indexical) sense cannot be a single way of presenting such different denotations to a mind.²⁶ Clearly we are talking about different modes of presentation of different denotations. The idea that the sense of the term that denotes the customary sense of “Masterpiece(Opus 132)” does not introduce a further sense beyond that customary sense of the sentence is really indefensible. But I will follow out the idea in the context of our principles to show where it leads.

Peacocke does not discuss the senses of the canonical sense-names that he introduces into the meta-language. But it is obvious that on Fregean theory, they have to have senses. They express different ways of thinking about senses than, say, definite descriptions or other names of

the senses that we might whimsically introduce. We should be able to form canonical names of the senses expressed by these first-level canonical names. Thus it would seem that we can regard the expression “<<Masterpiece(Opus 132)>>” as denoting the canonical sense of the meta-linguistic expression “<Masterpiece(Opus 132)>”. On the one-level anti-hierarchy view, the sense denoted by the first expression will be the same as the sense denoted by the second.

I think that ultimately one should prefer not to iterate the pointed brackets. Like quotation marks, they do not create an extensional context within them. At the bottom level of the hierarchy there is no need to reveal structure. So single non-iterated occurrences of pointed brackets just yield a finite number of first-level, structurally simple canonical names. But in order to capture the structure of the hierarchy one will need devices that reveal more extensional structure. For this purpose, it is preferable to use the functional expression “C” that I introduced earlier. Such use will allow the quantifications into embedded contexts that I discussed earlier. Thus, “C(<Opus 132>)” is a canonical name of the canonical sense that determines the sense denoted by “<Opus 132>”. (Of course, “Opus 132” does not occur as a name in extensional position within the pointed brackets.) In the argument that follows, I will use iterated pointed brackets, just to stay within Peacocke’s terminology. I will not need to make use of the upward functional structure of the sense hierarchy.

If one is to avoid, in the truth-theoretic meta-language, exactly the same hierarchy of senses that Frege makes use of to account for iterated contexts, one must claim that for canonical names of senses, the senses of the names (which translate expressions that in the object-language occur in that-clauses) are identical with the senses that they denote. This view captures in the meta-language the way that expressions in that-clauses operate in the object-language—according to the view that collapses the hierarchy to one level (Peacocke’s view): The expressions both

denote and express the expressions' customary senses.

Such a view may seem to make only a small concession to the broadly Russellian idea that names have denotation but no (new) sense, or that sense is identical with denotation. The concession is for special names—canonical names of senses. It holds that the sense and denotation of a canonical name for a sense are the same. The concession appears to be fairly close to Russell's restricted view of "logically proper" names, which named universals that we "grasp" by acquaintance. In fact, this seemingly small concession leads to very serious trouble.

Here is a further argument that is relevant to understanding the difficulties I raised for a meta-language that specifies senses (or translates object-language expressions into the meta-language), gives a truth theory for Method I object-languages, and attempts to avoid the hierarchy.²⁷ The argument is that either the meta-language is committed to a hierarchy of senses after all, or it falls into a different version of collapsing sense and denotation.

For simplicity, I will carry out the argument on the sense named by "<Opus 132>", rather than on the sense named by "<Masterpiece(Opus 132)>". This will circumvent the grammatical issue discussed earlier. I think that I have shown that that issue is not fundamental. But just to keep matters simple, I will avoid it in this argument from the beginning.

Let "a" be a dummy singular term. Suppose, according to the view that collapses the hierarchy and that makes the concession to Russell:

$$a) \langle\langle\text{Opus 132}\rangle\rangle = \langle\text{Opus 132}\rangle.$$

Then

$$b) \langle = (a, \langle\text{Opus 132}\rangle) \rangle = \langle = \rangle^{\langle a \rangle^{\langle\langle\text{Opus 132}\rangle\rangle}}.$$

b) follows from a) by the functional decomposability of senses. b) says that the sense of the meta-linguistic sentence " $\langle = (a, \langle\text{Opus 132}\rangle) \rangle$ "--or to write it differently, the sentence " $\langle a = \langle\text{Opus 132}\rangle \rangle$ "

132>)"--is the same as the result of functionally decomposing that sense into the senses of the component expressions.

$$c) \langle = \rangle^{\langle a \rangle^{\langle \text{Opus 132} \rangle}} = \langle = \rangle^{\langle a \rangle^{\langle \text{Opus 132} \rangle}}$$

c) follows from b) and a) by substitutivity of identity (here applied in a context that clearly does not translate or otherwise formalize a natural-language oblique context).²⁸

$$d) \langle = \rangle^{\langle a \rangle^{\langle \text{Opus 132} \rangle}} = \langle = \rangle^{\langle a, \text{Opus 132} \rangle}$$

d) follows from c) by compositionality of senses.

$$e) \langle = \rangle^{\langle a, \langle \text{Opus 132} \rangle \rangle} = \langle = \rangle^{\langle a, \text{Opus 132} \rangle}$$

e) follows from b)-d) by transitivity of identity.

$$f) \langle = \rangle^{\langle a, \langle \text{Opus 132} \rangle \rangle} \longleftrightarrow \langle = \rangle^{\langle a, \text{Opus 132} \rangle}, \quad \text{or equivalently}$$

$$f') a = \langle \text{Opus 132} \rangle \longleftrightarrow a = \text{Opus 132}$$

f), f') follow from e) by the principle that sentences have the same truth value if their senses are the same.

$$g) \text{Opus 132} = \langle \text{Opus 132} \rangle$$

g) follows from e) and f) by substituting "Opus 132" for "a", the truth of self-identities, and propositional calculus. (Cf. note 21.)

So if one makes what initially may have seemed to be a small concession to a Russellian conception of canonical sense-names in order to avoid in the meta-language the sense-hierarchy, and if one maintains very plausible general principles about sense (especially plausible if one thinks of sense—as one should—as thought content), the distinction between sense and denotation for singular terms collapses across the board. The argument assumes only the functional compositionality and decomposability of senses, substitutivity of identity in contexts that do not translate or formalize natural-language oblique contexts, and the principle that the senses (read:

cognitive values or thought contents) of sentences determine their truth values.

All of the principles, except for the collapse of the sense and denotation for canonical sense-names, are Fregean principles. All are attractive, and can be independently motivated. I believe that they are very plausible components in any theory of the structure of thought. Substitutivity of identity is necessary in a meta-language in which a truth-conditional semantics is to be carried out. Moreover, in the argument, substitution is applied only in contexts that do not involve a propositional attitude predicate. The argument is replicable at any higher level of the hierarchy. So it re-applies to any view that collapses doubly indirect senses and indirect senses. Analogs will apply for other expressions besides names or singular terms. So an analog will apply to senses of predicate expressions.

One could just concede that a sense hierarchy does appear in the meta-language, but that it is not used in accounting for oblique contexts in the object-language, allegedly the natural language. This move seems to me ad hoc. It is completely unobvious why one should not be able to translate the hierarchy of sense names (necessarily present in the meta-language, given the principles I have articulated, if it is to give a truth-theory for the object-language) back into the object-language. (And cf. note 15.) Again, bear in mind that this hierarchy of sense-names is simply a system of ways of thinking or cognitive perspectives. How could one deny that we are capable of thinking in these ways through the medium of natural language? I know of no bar to compositional translation of these (non-semantic) expressions between object-language (or natural language) and meta-language.

There are deniable assumptions that would enable one to avoid these difficulties. But all the relevant assumptions are Fregean, well motivated, and at the very least plausible. They are especially plausible, it seems to me, as applied to a theory of thought content.

Frege's approach to the language of ascription of propositional attitudes, and to thought about propositional attitudes, leaves out a lot. Frege ignores the fact that in many ascriptions we do not care about the exact way that an individual thinks about a subject matter. Sometimes we simply specify the subject matter. In ascriptions we often specify the subject matter of a person's thought in ways that we do not believe correspond at all to the way the person thought about the subject matter. In such cases, we "quantify into" that-clause constructions.

Frege's passing over this important and widespread phenomenon derives from his interest in an ideal scientific language for attributing propositional attitudes. An ideal language for this purpose would center on the nature of the attributed attitudes, fully specifying that nature—specifying the way that the relevant person actually thought. He was not interested in natural language ascriptions per se. He was especially not interested in attitude ascriptions insofar as the ascriptions are governed by pragmatic or contextual communicative concerns. He was interested in an ideal language for attributing attitudes in order to specify what the attitudes are. This involves specifying the representational perspective of the individual with the attitudes. I believe that this is not merely an old-fashioned interest. It remains an interest of scientific psychology and of any common sense attribution that centers on conveying the nature of the attitude as fully and accurately as possible.

Frege's approach also does not address attribution of de re attitudes. Here I believe that his account of thought and sense is inadequate to explain the nature of the attitudes themselves. There are elements of de re propositional attitudes that are not timeless thought contents.²⁹

Nevertheless, I believe that Frege's structural approach is, as far as it goes and given its aims, fundamentally on the right track. When one focuses not on linguistic meaning but on

thought expressed and attributed by language—and one thinks of thought content as perspective on or way of thinking about a subject matter—, the insight and power in Frege’s approach tend to emerge. In the context of those aims, I believe that the postulation of a hierarchy of cognitive modes of presentation, or of ways of thinking, or of perspectives that are associated with propositional attitude attributions, is tenable and arguably superior to theoretical alternatives.

The hierarchy is forced by basic principles that plausibly apply to thought content, propositional perspectives, ways of thinking. The relevant principles are those of the functional compositionality and decomposability of senses—of representational thought contents—, the functional dependence of truth-value on the denotations of constituent linguistic parts (or the determinata of constituent representational contents), and the functional (determination) relation between senses and denotations. Again one should think of senses as ways of thinking, or perspectives that mark aspects of perspectival abilities, or representational contents. I believe that in view of the way that senses are formed and denoted through canonical names, the hierarchy is far from the bugbear that most philosophers have presented it as being. In itself, it is simply another example of the functional productivity of language and thought.

The sense hierarchy is, to my mind, particularly interesting in that it provides a new example of how subject matter (here, senses or thought-component concepts) can help determine the way we think about a subject matter (here, the canonical senses or concepts that determine them)—even as the way we think about the subject matter also representationally determines the subject matter. The canonical senses or concepts take a cue from Russell: understanding them requires understanding their denotations. But the sense hierarchy remains fundamentally Fregean: a cognitive perspective is partial and is to be distinguished from what it is a perspective on.

Tyler Burge

Appendix I – Direct Pressure on Method I

In both “Frege and the Hierarchy” and in the text, I discuss Method I mainly by considering its reliance on Method II for a semantical theory. I show how a hierarchy is induced for a Method II meta-language for Method I. Here I want to discuss a more direct argument that suggests that on plausible principles, Method I itself is directly committed to a hierarchy.

It will be recalled that Method one allows systematic ambiguity of terms as between non-oblique contexts and oblique contexts. The point at issue is how far ambiguity extends. One might believe that there is only one or two levels of sense that are associated with embeddings of oblique contexts. In this Appendix I want to give and discuss an argument for the view that Method I should be directly committed to an infinite hierarchy of senses to account for embedded contexts. The argument will be very similar to the last of the arguments that I give in the text, with one important alteration.

A term like “Opus 132” will be treated on Method I as having a customary sense when it occurs in a non-oblique context—a context like “Opus 132 = Opus 132”. It will be treated as denoting this customary sense in an oblique context. The question at issue is whether there is a further sense, beyond the customary sense, expressed in ordinary unembedded oblique contexts. A one-level theory of sense gives a negative answer to this question. The argument that I give will be directed against this answer. But substantially the same argument can be given against any theory that stops the level of sense (for a given level of embedding) at a finite level. Thus the argument can be adapted to be directed against a Method I theory that holds that there are only two levels of sense, the customary sense of an expression, and a further sense expressed by the expression in an oblique context (where the expression denotes its customary sense). Such a

theory would deny that a yet further (third) level is needed to deal with what sense is expressed in an embedding of an oblique context within a further oblique context. Similarly, the argument can be adapted to be directed against a Method I theory that holds that there are only three levels of sense.

To make the argument easy to read, let us introduce a notation for the phrase “the sense of expression E in Context C ”. Let this expression be abbreviated by the two-place functional expression “ S ”. Let “ N ” denote the class of non-oblique contexts. For example, “ N ” would denote a class that contains both singular term positions in the sentence “Opus 132 = Opus 132” standing alone (where “=” is read as “is identical with”). Let “ $O-1$ ” denote the class of oblique contexts that are not embedded in a further oblique context. For example, “ $O-1$ ” would denote a class that contains the position of “Opus 132” (occurring obliquely) in the sentence “Bela believes that Opus 132 is a masterpiece”. I use pointed brackets (“ $\langle _ \rangle$ ”) as I do in the text, except that I include a marker for the context in which the term within the brackets is to be taken as occurring. Thus “ $\langle \text{Opus 132, } O-1 \rangle$ ” is a canonical name for the sense of the expression “Opus 132”, as “Opus 132” occurs in an unembedded context; “ $\langle \text{Opus 132, } N \rangle$ ” is a canonical name for the sense of the expression “Opus 132” as “Opus 132” occurs in a non-oblique context.

Here is the argument:

$$1) S(\text{“Opus 132”, } O-1) = S(\text{“Opus 132”, } N) = \langle \text{Opus 132, } O-1 \rangle = \langle \text{Opus 132, } N \rangle .$$

1) simply records the one-level-of-sense view of the semantics of a Method I language. We will not use the latter two identities in 1) in the argument. I record them simply for clarity.

$$2) S(\text{“Opus 132 = Opus 132”, } N) = S(\text{“Opus 132”, } N) \wedge S(\text{“=”, } N) \wedge S(\text{“Opus 132”, } N).$$

2) says, in effect, that the sense of a sentence occurring in a non-oblique context and asserting the self-identity of Opus 132 is decomposable into the senses that the component parts of the

sentence express in a non-oblique context.

$$3) S(\text{"Opus 132"}, N) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, N) = \\ S(\text{"Opus 132"}, N) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-1).$$

3) follows from 2) and 1) by substitutivity of identity. Note that the substitution does not occur in an oblique context, or in a context that formalizes an oblique context.

Let "a" be a name of the customary sense of "Opus 132", and let "a" be capable of occurring in non-oblique contexts. Let "a" express (in non-oblique contexts) any sense one likes that is compatible with these explanations of its denotation or reference and of its grammatical behavior. Then

$$4) a = S(\text{"Opus 132"}, N) = S(\text{"Opus 132"}, O-1).$$

4) follows from the explanation of "a" and 1).

$$5) S(\text{"Opus 132"}, N) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-1) = \\ S(\text{"Opus 132"}, N) \wedge S(\text{"="}, N) \wedge a.$$

5) follows from 4) and 3) by the substitutivity of identity.

$$6) S(\text{"Opus 132"}, N) \wedge S(\text{"="}, N) \wedge a \\ S(\text{"Opus 132 = a"}, N)$$

6) follows from 5) by the compositionality of sense. The sense (or sense-proposition or thought) composed appropriately of the senses of the semantically relevant parts of a sentence is identical with the sense (or sense-proposition or thought) expressed by the whole sentence.

$$7) S(\text{"Opus 132 = Opus 132"}, N) = S(\text{"Opus 132 = a"}, N).$$

7) follows from 2)-6) by the transitivity of identity.

$$8) \text{Opus 132} = a.$$

8) follows from 7) by the truth of self-identities and the principle that sentences with the same sense are materially equivalent.

8) is absurd. It says in effect that Opus 132 is identical with the customary sense of “Opus 132”. Since the argument can apply to any expression and since it can apply to any account that stops at a finite level of senses expressed in successively embedded oblique contexts, it shows that in a Method I language, sense and denotation (or reference) collapse if the principles relied upon in the argument are accepted.

The principle premises used in the argument are again the decomposability of sense, the compositionality of sense, and the principle that sentences with the same sense have the same truth value. There are two further background assumptions used in the the argument.

One is that it is permissible in the Method I language to use functional terms (containing “S”) that specify the senses of expressions relative to contexts, where the specifications are in accord with a given theory’s account of what those senses are. Thus one is producing functional specifications of senses with complex singular terms that can occur in non-oblique contexts. I cannot see any ground to reject this assumption.

The other assumption is that it is permissible in the Method I language to introduce a name like “a” that can denote a sense but occur in a non-oblique context. I believe that this assumption is really dispensable in favor of the first assumption. It is just that the argument would require more apparatus to deal with embedding of quotation marks within quotation marks, or some analogous system for denoting expressions that denote expressions. I cannot see that there is any principled reason for rejecting either of the assumptions.

I have claimed that the argument just given can be adapted to show that attempt to stop the hierarchy at a finite level, in accounts of embedded oblique contexts, will on the relevant principles collapse the distinction between sense and denotation. Since these matters are complex, I will illustrate how the argument works at one higher level—against a view of Method I

languages according to which customary sense and indirect sense are distinguished, but indirect sense is identical with doubly indirect sense. That is, the following argument reduces to absurdity the view that in a Method I language, there are only two levels of sense. It reduces to absurdity the view that the sense of an expression occurring obliquely in an embedded oblique context is the same as its sense in an unembedded oblique context (though distinct from its customary sense, which is denoted in an unembedded oblique context).

I will use iterated quotation marks that embed quotation marks, but I think it clear that these can be dispensed with, without affecting the argument. Here is the argument:

$$1') S(\text{"Opus 132"}, O-2) = S(\text{"Opus 132"}, O-1)$$

$$1a') S(\text{"Opus 132"}, N) \text{ is not identical with } S(\text{"Opus 132"}, O-1).$$

$$2') S(\text{"S(\text{"Opus 132"}, O-1) = S(\text{"Opus 132"}, O-1)"}, N) =$$

$$S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-1)$$

2') follows from the decomposability of the sense of a sentence asserting that the indirect sense of "Opus 132" is self-identical.

$$3') S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-1) =$$

$$S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-2)$$

3') follows from 2') and 1') by substitutivity of identity.

Let "b" be a name of the sense of "Opus 132", as it occurs in non-embedded oblique contexts. That is, "b" names the indirect sense of "Opus 132". It names $S(\text{"Opus 132"}, O-1)$.

Let "b" be capable of occurring in non-oblique contexts. Let "b" express (in non-oblique contexts) any sense one likes that is compatible with these explanations of its denotation or reference and of its grammatical behavior. Then

$$4') b = S(\text{"Opus 132"}, O-1) = S(\text{"Opus 132"}, O-2).$$

4') follows from the explanation of "b" together with 1').

$$5') S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge S(\text{"Opus 132"}, O-2) = \\ S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge b.$$

5') follows from 3') and 4') by substitutivity of identity.

In an unembedded oblique context "Opus 132" denotes only its customary sense and expresses its indirect sense ($S(\text{"Opus 132"}, O-1)$). Let us introduce a term "B" that, in a non-oblique context, expresses the sense that "Opus 132" expresses in an unembedded oblique context and denotes the sense (the customary sense of "Opus 132") that "Opus 132" denotes in that same unembedded oblique context. So "B" denotes $S(\text{"Opus 132"}, N)$. That is, "B" has the same semantical characteristics in a non-oblique context that "Opus 132" does in an unembedded oblique context. Then

$$6') S(\text{"Opus 132"}, O-1) \wedge S(\text{"="}, N) \wedge b = \\ S(\text{"B = b"}, N).$$

6') follows from 5') by the compositionality of sense and the explanation of the term "B".

$$7') S(\text{"S(\text{"Opus 132"}, O-1) = S(\text{"Opus 132"}, O-1)"}, N) = S(\text{"B = b"}, N).$$

7') follows from 2')-6') by the transitivity of identity.

$$8') B = b.$$

8') follows from 7') by the principle that sentences (standing alone) that have the same sense also have the same truth-value.

8') contradicts 1a'). Thus again, on plausible principles, a Method I language will collapse the hierarchy altogether if it identifies an expression's sense with its denotation at any level of embedding of oblique contexts.

The plausible principles are the decomposability of sense, the compositionality of sense,

and the principle that sentences (standing alone) with the same sense have the same truth-value.

As in the first argument in this Appendix, there is one additional assumption: We can introduce into Method I languages names or descriptions that in non-oblique contexts express the senses and denote the denotations that other expressions express and denote in oblique contexts. I see nothing in the conception of Method I languages, or in the conception of Method I languages that I am criticizing, that prevents one from introducing such expressions.

What this argument and the first argument in this Appendix show is that the substitutivity of co-extensive expressions that differentiates Method II from Method I languages, and which makes Method II languages superior as a basis for giving a truth theory, is not fundamental to yielding the arguments for the hierarchy. More basic is having expressions that can appear in non-oblique positions but which denote mimic the semantical behavior of expressions that appear in that-clauses. They must denote and (after the first-level) express the senses of expressions that appear in that-clauses. In a Method I language in which a term can occur either non-obliquely or obliquely at various levels of embedding, the argument simply requires that there be other expressions that have the same semantical characteristics—have the same sense and denotation—as occurrences of terms in oblique position. A scientific language that has any chance of describing the structure of sense or thought-content as expressed in natural language must have such terms.

Thus on plausible principles, Method I languages generate a hierarchy just as surely as Method II languages do.

Appendix II—Existential Generalization on Embedded Positions

In the main text, I discussed existential generalization on

(2) Igor believes that Bela believes that Opus 132 is a masterpiece.

(2) entails

(2Exp) There is something, namely that Opus 132 is a masterpiece, that Igor believes that Bela believes,

which entails

(2EG) There is something that Igor believes that Bela believes.

I formalized (2) as

(2a) $\text{Believes}(I, \langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} C(\langle \text{Opus 132 is a masterpiece} \rangle))$.

I formalized (2Exp) as

(EXP) $(\exists y)(y = \langle \text{Opus 132 is a masterpiece} \rangle \ \& \ \text{Believes}(\text{Igor}, \langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} C(y)))$.

(2a) also formally entails

(EG) $(\exists y)(\text{Believes}(\text{Igor}, \langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} C(y)))$.

I believe that (EG) formalizes (2EG).

It is worth noting that (2a) also formally entails

(EG') $(\exists z)(\text{Believes}(\text{Igor}, \langle \text{Bela} \rangle^{\wedge} \langle \text{Believes} \rangle^{\wedge} z))$.

Here it may be tempting to raise an objection to the account that I have given. One might be tempted to hold that in view of the fact that (EG) and (EG') both follow from (2a), it is a consequence of (2a)'s formalizing (2) that I am committed to (2)'s entailing that Igor believes Bela believes at least two things.

I believe that this objection is mistaken. This Appendix will explain why. The larger point

of the explanation is to elicit a better understanding of the hierarchical account, and to explore its relation to English. I believe that (EG) and not (EG') formalizes (2EG).

As noted, (EG) and (EG') are both entailed by (2a). It is worth noting that there are yet further existential generalizations in the offing. In the first place, there will be more existential generalizations (EG''), (EG''')... with further embeddings ("Arnold believes that Igor believes that Bela believes that Opus 132 is a masterpiece"...), and further iterations of "C" in the formalizations. In the second place, there are positions in the structured root proposition-name that denote senses that are components of the denoted proposition or thought content (e.g. the position of "<Opus 132>" in the structured "<Opus 132 Masterpiece>", which really has the form "<Opus 132>^<Masterpiece>"). From these positions one will have even more quantifications that follow from the initial formalization (2a).

I think that the first of the quantifications, (EG), is what formalizes the English quantification

(2EG) There is something that Igor believes that Bela believes.

Only (EG) captures the inference to (2EG) from

(2) Igor believes that Bela believes that Opus 132 is a masterpiece.

I believe that I can say why (EG) and not (EG') formalizes (2EG). I will begin by explaining why (EG) and not (EG') tracks the inference from (2) to (2EG). I will come back later to discussing the interpretation of (EG').

The reason begins with this observation: (2EG) follows from (2) inasmuch as Igor is characterized in (2) as believing that Bela believes a certain proposition or thought content (namely, that Opus 132 is a masterpiece). That is, we got (2EG) by first reasoning from (2) to

(2Exp) There is something, namely that Opus 132 is a masterpiece, that Igor believes that

Bela believes

and then dropping the conjunct that consists in “namely that Opus 132 is a masterpiece”.

Perhaps it is possible for Igor to be so odd that he believes that Bela believes things other than propositions. Perhaps--although this is at best questionable--Igor could believe that Bela believes a fire engine, or a stone. Perhaps (again questionable) Igor could believe that Bela believes the sense of a singular term, a sense which is not a thought content or proposition. Bela cannot in fact believe a fire engine or the sense of a singular term. What might be disputed is whether Igor could believe he can. It would be easier on us all if we could just declare that no one can believe that anyone believes something other than a proposition or a thought content. Perhaps then there would be fewer entailments to account for! But let us proceed on the assumption that it is possible for someone erroneously to believe that someone believes something other than a proposition or thought content.

However, the only thing that it is guaranteed that Igor believes that Bela believes, given the truth of the sentence (2), is a proposition, or thought content-- namely, the thought content that Opus 132 is a masterpiece. There are not two things that Igor believes Bela believes in virtue of the truth of (2).

I think that the same is true of (2a). (2a) does not entail anything that commits us to saying that Igor believes that Bela has another belief besides the belief that Opus 132 is a masterpiece. The only relevant term that denotes a proposition or thought content that Igor could believe Bela believes in my formalization (2a) is "<Opus 132 is a masterpiece>"--*not* "C(<Opus 132 is a masterpiece>)". The latter does not denote a proposition or thought content. It denotes the sense of the canonical name "<Opus 132 is a masterpiece>". (This latter canonical name denotes the customary sense of “Opus 132 is a masterpiece”.) So, even if we grant that it is

possible, it would be *very* odd of Igor to believe that a non-proposition is believed by Bela. And it is certainly not guaranteed or implied by (2) or (2a) that he believes that Bela believes any such thing. So if we export “C(<Opus 132 is a masterpiece>)”, we are not exporting a name of some proposition that Igor believes Bela believes. It is wrong to think that the canonical name (or designator) names anything that Igor believes that Bela believes. I will come back to how we should read such exportations.

Both in English grammar and in formalizations of English, one should keep track of the level of entity (and within sense-levels, the type of entity—e.g. sense of a singular term, sense of a sentence, and so on) that is being quantified over. One has to do some of this anyway to distinguish quantifying into that-clauses (in effect, all the way down to the bottom level of reference or denotation) from quantifying onto them (quantifying over a sense or mode of presentation). Thus in a full theory of quantification, which I am not trying to give here, one should probably mark the variable (here “y”) that “C” applies to as a variable that goes with the name of a proposition or thought content-- as distinguished from the name of a customary sense of a name, and as distinguished from the name of a customary sense of a predicate.³⁰

We have the following situation: (2EG) follows from (2). (2a) formalizes (2). Both (EG) and (EG’) follow from (2a). So both (EG) and (EG’) should follow from (2). But (EG’) does not generalize on a second belief of Igor’s, a belief over and above the belief that Opus 132 is a masterpiece (contrary to the tempting objection that we began with). So generalization on the position of “C(<Opus 132 is a masterpiece>)” in (2a) is not what is going on in the English (2EG). (2EG) is not true in virtue of a generalization on “C(<Opus 132 is a masterpiece>)”. For this would take the relevant singular basis for the existential generalization to be the sense of a singular term, not a thought content—the sense of a sentence.

I will stipulate (since I get to make up the example) that Igor does not believe that Bela believes the sense of a singular term. Igor attributes only propositional beliefs to Bela.

Another way of putting all this is that we cannot get to a formalization of (2EG) by exporting “C(<Opus 132 is a masterpiece>)” from (2a) to get

(Exp’) $(\exists z)(z = C(\langle \text{Opus 132 is a masterpiece} \rangle) \ \& \ \text{Believes}(\text{Igor}, \text{Bela}) \wedge \langle \text{Believes} \rangle^z)$,

taking this to be a formalization of

(Weirdo) There is something, namely the canonical sense of the name “that Opus 132 is a masterpiece”, that Igor believes that Bela believes

and then dropping the first conjunct of the existential generalization to get

(EG’) $(\exists z)(\text{Believes}(\text{Igor}, \langle \text{Bela} \rangle \wedge \langle \text{Believes} \rangle^z))$,

taking this to formalize

(2EG) There is something that Igor believes that Bela believes.

There is no analogy here to our first exporting “that Opus 132 is a masterpiece” from (2), then our formalizing this exportation by (EXP), and then our simplifying (EXP) to (EG).

In view of the fact that the “z” in (EG’) does not trace back to the name of a proposition, it is not made true by anything Igor believes that Bela believes. Moreover, (Weirdo) is not a correct reading of (EG’), since “the canonical sense of the name “that Opus 132 is a masterpiece”” is a definite description and “C(<Opus 132 is a masterpiece>)” is a canonical name or canonical designator—not a definite description. Our canonical names represent specifically oblique occurrences of expressions in natural language, most prominently such occurrences in ordinary or in embedded that-clauses. “C” specifically marks levels of embedding in canonical names of modes of presentation or thought content.

Just to fix this last point, let us consider formalizations of

(2Weird) Igor believes that Bela believes the fire engine

(2Weirdo) Igor believes that Bela believes the canonical sense of the name “that Opus 132 is a masterpiece”.

I believe that one can take each of these as having a quantifying-in reading and an oblique occurrence reading, if they have any readings at all. Let us focus on the oblique readings. The propositions that Igor believes are expressed by the sentences

(2Weird -) Bela believes the fire engine

(2Weirdo -) Bela believes the canonical sense of “that Opus 132 is a masterpiece”,

where the direct objects occur obliquely. Suppose that the terms in direct-object position denote their customary senses. Suppose that we allow first-level canonical names to formalize those occurrences and to denote the customary senses:

<the fire engine>

<the canonical sense of “that Opus 132 is a masterpiece”>

Then in formalizations of (2Weird) and (2Weirdo) we must introduce canonical names that denote the senses of these canonical names, as those senses are expressed in (2Weird -) and (2Weirdo -). Thus we formalize (2Weird) and (2Weirdo) as

(2Weird - f) Believes(Igor, <Bela>^<Believes>^C(<the fire engine>))

(2Weirdo - f) Believes(Igor, , <Bela>^Believes>^C(<the canonical sense of “that Opus 132 is a masterpiece”>))

I believe that these formalizations convincingly suggest two things. One is that if one keeps track of whether a canonical name formed with “C” derives from a root canonical name of a proposition or a root canonical name of a sense other than a proposition, one will never get formalizations in embedded cases that confuse the two, or that are inspecific as between the two.

The other thing suggested by the formalizations is that canonical names enter initially as

formalizations of occurrences of other expressions of the natural language that occur obliquely. On Method II, which we are following, no canonical name can itself occur obliquely. The first-level canonical names (which I have been formalizing with pointy brackets) will formalize all root weird occurrences. (Cf. the formalizations of (2Weird -) and (2Weirdo -).) “C” comes into play only in formalizations of embeddings—usually embedded that-clauses, but perhaps also embedded attributions of Weird beliefs that are not propositional. I believe that in no case is it true that such formalizations allow a canonical name which includes “C” to formalize a root direct object of an embedded occurrence of “believe” —whether the object denotes a proposition or not. Thus (EG’), understood as quantifying onto a place that had contained a canonical name that includes “C” does not formalize any sentence like (2Weird) or (2Weirdo). I conjecture that wide-scope quantifications that formalize natural language quantifications on the direct object of the final or root direct object always leave the “C” (possibly a string of iterated “C”’s) in place (i.e. not exported), as (EG) does. I will soon discuss how to read back into English formalizations that do export canonical names that include “C”.

There are surely further things to be said of

(2EG) There is something that Igor believes that Bela believes.

(2EG) should follow even from Igor’s believing that Bela believes a fire engine or believes the sense of a singular term. Our formalizations follow this course. For example, (EG), which does follow from (2Weird) and (2Weirdo), is a formalization of (2EG)—though as I said, I believe that the English grammar as well as the formal theory should leave a trace of the fact that here (2EG) derives from an attribution by Igor of a non-propositional belief to Bela.

The key point for present purposes is that (EG’) should not be the formalization of the generalization (2EG) that follows even from (2Weird) and (2Weirdo). It is (EG) that is the

relevant formalization of the generalization (2EG) that follows from (2Weird) and (2 Weirdo).

So an issue has become: what *other* English sentence might be formalized by (EG')?

Let us first consider where the existential generalization in (EG') came from. Let us think about how we would read the exportation of "C(<Opus 132 is a masterpiece>)" from (2a). The formalized exportation is

$(Exp') (Ez)(z = C(\langle \text{Opus 132 is a masterpiece} \rangle) \ \& \ \text{Believes}(\text{Igor}, \langle \text{Bela} \rangle \wedge \langle \text{Believes} \rangle \wedge z))$.

This can be glossed in English:

(Engl-Exp') There is a (canonical indirect-sense-level) way of thinking about the thought that Opus 132 is a masterpiece that Igor's belief utilizes in attributing that thought (that Opus 132 is a masterpiece) to Bela as a belief.

One could get from this reading down to this English gloss on (EG'):

(Engl-EG') There is a way of thinking about a thought that Igor's belief utilizes in attributing a thought to Bela as a belief.

or more briefly,

(Engl'-EG') There is something that Igor's belief utilizes in attributing a belief to Bela.

Could this be the English sentence that (EG') formalizes?

One might protest that these glosses have "extra" words--"way of thinking", "thought", "utilizes", "attributing" that have no counterparts in the formalizing sentences (EG) or (EG').

The fact that there isn't a *smooth* reading in English of (EG') is not, I think, a difficulty.

I am inclined to believe that the situation with the English in this case is broadly analogous (but perhaps not ultimately as deeply interesting) as the problem about expressing second-order quantification in English. From the natural formalization of "Sally found Bill"-- $F(s, b)$ --, one can infer

$(EF)F(s, b)$.

One can put this in English as something like

Sally bore some relation to Bill,

This sounds more “English” than “Sally F-ed Bill, for some F”.

One might reply “But then the verb is “bore””, or “But then the relation is *bore*”. Well, yes. But it does not follow that we cannot do enough meta-talk about how to understand the English that it is clear that we are understanding words like “bore” as syncategorematic, or as indicators of the second-order quantification. Surely one can express the second-order quantification in English. So it seems that we should be able to explain an understanding of the English according to which we have a second-order quantification over a two-place relation (in “Bill bore some relation to Sally”) rather than a first-order quantification over an argument or relatum in a three-place relation *bore*. I think that one *can* engage in such meta-talk and that English can express the second-order locution, though it does not do so smoothly on its surface.

In the case of the “excess” quantifications in the embedded that-clause cases (of which (Engl’-EG’) is an example), it is not a matter of second-order quantification. But I think getting the readings in English of the formalizations will force “extra” words into the English in an analogous way. Here the syncategorematic words are ultimately “utilizes” and “attributes”.

My conjecture is that any view that both treats that-clauses as singular terms with structure and that treats that-clauses in natural language as having oblique positions (i.e. any view that has any chance of being correct as an account of how English can specify propositional attitudes in terms of their representational thought contents or modes of presentation) will throw up quantifications from the formalizations that cannot be smoothly read back into English, but which are true and are (with circumlocution) credibly expressible in English.

Footnotes

1. This Postscript benefitted from two sessions in which I presented the ideas in it to the UCLA philosophy of language workshop in Winter 2004. Comments by Tony Anderson, David Kaplan, Nathan Salmon, Philippe Schlenker, and Terry Parsons led to improvements.
2. Rudolf Carnap, Meaning and Necessity, (Chicago, Chicago University Press, 1967; originally published 1947), p. 131.
3. Michael Dummett, Frege: Philosophy of Language (Cambridge, Massachusetts; Harvard University Press, 2nd edition, 1981), p. 267ff.
4. Terry Parsons, “Frege’s Hierarchies of Indirect Sense and the Paradox of Analysis”, Midwest Studies in Philosophy, vol. 6 (1981), pp. 37-57. I say “Early Terry Parsons” to distinguish this view from the view of his later paper, cited in notes 11 and 14.
5. Donald Davidson, “Theories of Meaning and Learnable Languages”, reprinted in Inquiries Into Truth and Interpretation (New York, Oxford University Press, 2nd edition, 2001; originally published 1965). Strictly speaking, Davidson claimed that Church’s language of the Logic of Sense and Denotation is unlearnable. I think that this point is doubtful, even applied to Church’s particular language. (The issue depends on how Church’s subscripts are related to a prior understanding of an upwardly-functionally-determined hierarchy of canonical concepts.) I believe that Davidson and many others have taken the argument to apply to any language that invokes a hierarchy of sense. Dummett does, for example, in Frege: Philosophy of Language, op. cit., p. 167. What is clear, I think, is that the version of the hierarchy I outline here is not vulnerable to Davidson’s objection.
6. Alonzo Church, “A Formulation of the Logic of Sense and Denotation” in The Collected Works of Alonzo Church, Burge et. al. editors (Cambridge, Massachusetts; MIT Press, 200?; originally published, 1951). CHECK.
7. This usage is, of course, distinct from Frege’s use of “concept” (Begriff). My usage is closer to that of Alonzo Church, “A Formulation of the Logic of Sense and Denotation”, op. cit..
8. S-denotation or determination is the relation between a sense and the entity that it uniquely determines. That entity is denoted by an expression that expresses the sense. The relation of s-denotation or determination is the relation that Church called the concept-of relation. Determination of an s-denotation or determinatum is the non-linguistic analog (between senses and entities rather than symbols and entities) of denotation. For Frege, and I think sometimes in fact, it is more basic than the relation a linguistic expression bears to its denotation by way of its sense.
9. Gottlob Frege, “On Sense and Reference”, op. cit., p. 66; p. 37 in the original.

10. For discussion of some of the complexities associated with this point, see my “Self-Reference and Translation,” Guenther-Reutte and Geunther, editors, Translation and Meaning (London: Duckworth and Company, Ltd., 1978), pp. 137-153.

11. This view represents a change of position, on my part, from the position in “Self-Reference and Translation”, ibid. I changed views relatively soon after writing that paper. At the time I wrote that paper, I was attempting to work out a Carnap-like position according to which words occurring obliquely in that-clauses denote words, construed as having definite meanings. I have subsequently given up that view in favor of the view maintained by Frege and Church—that such words normally denote abstracta that are not necessarily linguistic entities, in fact, normally not linguistic entities. I do continue to believe that the rebuttal of Church’s translation argument that I gave in “Self-Reference and Translation” both prevents the argument from being decisive, and indicates something important about the nature of quotation. The reason for maintaining the Church-Frege view about the denotation of words that occur obliquely is not Church’s translation argument, but simply that the Fregean view is more natural and provides a deeper account of what is really at issue in attributions of propositional attitudes—at least when those attributions are concerned with the natures of the attitudes being attributed.

12. The that-clause-forming expression “that” (like quotation marks) is not itself a functional expression. Nevertheless, the that-clause-forming expression “that” has many of the characteristics of a functional expression, especially in occurrences that embed other that-clauses. In a formal language, one needs names for the customary senses of ordinary first-level expressions. Then a functional expression like “C” can apply to these names, and then iterate. I claim that if one understands that-clauses, one has in effect implicitly understood such a functional expression.

13. I am comfortable in calling complex expressions formed in functional ways “names”. They are not descriptions. They are rigid. I think that they are similar to numerals in the base-ten system larger than “9”. But if one wants to require that names cannot have functional structure, or if these canonical expressions fail some other linguistic test for names, I am willing to call them “canonical designators”. The key facts about them are that they are non-descriptive, rigid expressions, whose denotation can be computationally determined. There is a further feature of canonical names of senses that I am about to articulate. This feature or principle distinguishes them from other canonical names or designators, including numerals.

14. If “C” syntactically applies to any singular expression that denotes a sense, it yields a name of the canonical sense that determines the sense denoted by the singular expression. But if “C” is applied to any expression other than a canonical name of a sense, the sense of the whole functional expression is not itself a canonical sense.

15. Gottlob Frege, Philosophical and Mathematical Correspondence, Gabriel et. al. editors (Chicago, University of Chicago Press, 1980), pp. 153-154.

16. Both of the objections that follow are given by Christopher Peacocke, “Entitlement, Self-Knowledge, and Conceptual Redeployment” Proceedings of the Aristotelian Society Vol 96 (1996), pp. 142-144. Cf. also his Being and Being Known (Oxford, Oxford University Press, 1999), pp. 245-262. He also makes an objection from an inference involving quantifying into that-clauses. I will not discuss this objection. I believe, however, that the hierarchical view is clearly at no disadvantage in dealing with quantifying in, as long as it is extended to allow for attributions of de re attitudes. Frege himself does not handle such cases. But the difficulties that his view faces do not derive from commitment to a hierarchy. I believe that his view needs to be supplemented to handle de re constructions and quantifying in, but I believe that these supplementations need not essentially affect the structures that we are discussing here.

17. A more nearly fully formalized language with these properties is set out by Terry Parsons, in “A Quasi-Fregean-Carnapian-EarlyKaplanian Semantics” forthcoming in a volume of essays honoring David Kaplan. Parsons’ up-arrow is essentially the same as my “C”. We came to the idea independently. I agree with his remark that the hierarchy is “a kind of epiphenomenon of the simple part at the basis of the hierarchy”—with the proviso that one needs, in addition to an understanding of the simple part at the basis, a conception of a functional notion for yielding canonical senses (expressed by his up-arrow and my “C”) to understand the cognitive content of the hierarchy. It is important that understanding canonical names at all levels depends on an antecedent understanding of the first-level customary senses. Parsons provides a detailed semantics in which he shows how to deal with inferences essentially similar to (2)-(4). He also discusses the argument of “Frege and the Hierarchy” and extends the argument in an illuminating way. I have some doubts about his account of variables relative to an assignment as being canonical names of the assigned objects. I believe, however, that this part of his view is not essential to formalizing relevant inferences. He seems to me to provide an elegant, insightful, and plausible formalization of a language committed to a hierarchy of senses, and a clear discussion of the motivation behind the principles that he appeals to. I think that these are clearly Fregean principles.

18. Daniel R. Boisvert and Christopher M. Lubbers, in “Frege’s Commitment to an Infinite Hierarchy of Senses”, Philosophical Papers, vol. 32 (2003), pp. 31-64, offer an argument from Fregean principles for the hierarchy. Their paper does not make reference to mine, but the argument that they give seems to me to be a fairly close variant of the one I give. They center directly (and I think illuminatingly) on the functional-compositional structure of senses, whether denoted or expressed. They also make a more explicit textual case than I did that the principles that rule out a hierarchy can be found in Frege. Frege’s commitment to the idea that one can recover the structure of thoughts from the structure of expressions expressing them occurs in various places. Cf. e.g. “Logic in Mathematics”, Posthumous Writings, op. cit., p. 207; Nachgelassene Schriften, op. cit., p. 224. The authors do not make direct use of the substitution principles that I appealed to in an object-language (for Method II) or meta-language (for Method I), as I did. And they do not discuss the Methods separately. But I explicitly associated the principles that I discuss with the functional-compositional structure of sense that they highlight. In one respect their argument is less general than my argument, or the extension of my argument that Parsons gives. It does not apply directly to an attempt to collapse the hierarchy

that appeals to only one level of sense. It shows on Fregean principles that if one is committed to two levels of sense, one is committed to the infinite hierarchy. They argue separately against a one-level theory. (I agree with some but not all of their objections to a one-level sense theory.) They also assume, appealing to Davidson's unlearnability argument, that Frege's commitment to the hierarchy of senses is untenable. As I have indicated, I believe that this assumption is very much mistaken.

19. Cf. "Entitlement, Self-Knowledge, and Conceptual Redeployment" *op. cit.*, pp. 153-157.

20. I continue to think that this objection to Method I raises interesting questions about the relation between a truth theory for a language and the structure of that language. I continue to think that the objection is correct. But I believe that there is an objection to taking Method I as a way of avoiding the hierarchy that is more direct. Such an objection works directly off the alleged senses and denotations of the parts, without using any "syntactical" premise about substitutivity. The argument that I gave in the original paper, using a premise about substitutivity, was envisioned as applicable to the meta-language that gives the truth theory for the Method I. But the last argument I give below can be adapted to apply directly to an object-language that follows Method I, without making use of any premise about substitutivity that Method I bars. Method I directly incurs a hierarchy on plausible principles. I outline this more direct argument in Appendix I—Direct Pressure on Method I. (It is probably advisable to read the Appendix after reading through the main text.)

21. I give two semi-formal arguments here. Both types of argument can be found in Terry Parsons, "A Quasi-Fregean-Carnapian-EarlyKaplanian Semantics", *op. cit.*, forthcoming. The first argument that I give is an application to Peacocke's language of a simplified variant on my 1979 argument. The second is an application of an extension of my 1979 argument by Parsons to singular terms. I think the argument generalizes further. My original argument focused on sentences within that-clauses. But the basic ideas are the same, as applied to any non-sentential expressions that have sense.

22. The notation "[^]" is a wave of the hand toward what will inevitably be an extremely complex account. The account must specify the various ways that one puts together names of complex senses of complex expressions, where the complex expressions have components of different syntactical categories. All that I assume here is that canonical names can be formed in such a way that the sense of the name of the sense of a complex expression is a function of the senses of the expressions that are components of the complex expression. The name-forming rules will make use of the syntactical rules governing the expressions with the senses named, and the ordering of the syntactical parts (with their senses) within the syntactical complex.

23. Peacocke's account assumes that the that-clause denotes as well as expresses its customary sense. I have mentioned earlier that I believe that this is an unattractive feature of the account—certainly out of keeping with the Fregean view that a sense determines a unique denotation. On his view the sense is a mode of presentation both of a sense and of a truth-value. Or if one rejects truth-values, the sense associated with "Opus 132" is a mode of presentation

both of a sense and of a string quartet.

24. There is, in my view, an unclarity about the logical syntax of “believes that p” in the object-language if one accepts the view that the sentence in the that-clause both denotes its customary sense and expresses its customary sense. Insofar as it expresses its customary sense, it is a sentence. Insofar as it denotes its customary sense, it is a term. I believe that this unclarity is an aspect of the fundamental problem. I believe that in actual fact the that-clause is a singular term, and “believes” is a relational predicate.

25. In “Frege on Truth” (1986) I maintain that the usual reasons for holding that Frege was mistaken in giving sentences and singular terms a single grammatical category for some purposes are not good ones. I do not think that there are deep philosophical reasons against Frege’s usage. But I prefer not to follow him on this point. So I do not ultimately rest my case here on siding with Frege against this complaint, even though I do not think that the complaint at all goes to the heart of the matter.

26. In my own view, certain ways of thinking are token applications, or abstractions from token applications. These do not determine their denotations by their nature. They depend on context for their successes. But for any given token application there is at most one denotation or referent. So even these “ways” of thinking are modes of presentation of an object (if any) that are individuated in such a way that a given way of thinking determines at most one denotation. Here way of thinking determines a unique referent if any, in the functional sense of “determines”. This is, I think, the sense of “determines” that is fundamental for understanding Frege and for understanding representation. For more on this, see my “Belief De Re”, The Journal of Philosophy, vol. 74 (1977), pp. 338-362 and “Five Theses on De Re States and Attitudes” in a forthcoming volume in honor of David Kaplan.

27. Again, the following argument is an application of an argument due to Terry Parsons, in ““A Quasi-Fregean-Carnapian-EarlyKaplanian Semantics”, op. cit. That argument is an extension of the argument I gave in “Frege and the Hierarchy”. Cf. note 14.

28. Note that here we apply substitutivity only in a non-oblique context. A variant of the argument that I am giving is applicable directly to Method I, see Appendix I. The style of argument that I am giving is not restricted to application to a Method II meta-language. But here I am illustrating a variant of the argument that I gave in “Frege and the Hierarchy” regarding a Method II truth theory for English or for a Method I language that is translational.

29. Cf. my “Belief De Re”, The Journal of Philosophy, vol. 74 (1977), pp. 338-362, and “Five Theses on De Re States and Attitudes”, forthcoming in a volume of essays in honor of David Kaplan.

30. Of course, Church’s “Logic of Sense and Denotation”, op. cit. has this feature.