

POSTSCRIPT to “Frege on Truth” CHECK’s

I would like to do four things in commenting retrospectively on this paper. First and second, I want to correct two mistakes in the paper. Third, I want to discuss critically a very different interpretation of Frege’s views on truth and logical consequence, associated with the second correction. Finally, I want to make some remarks about Frege’s view of the role of his conceptions of truth in expressing knowledge.

The first correction is easy to state. On page 141 [CHECK–this is of the original article] I took the propositional calculus to be the fundamental part of logic. It is true that the propositional calculus is the simplest part. I think, however, that the propositional calculus is not conceptually fundamental. I believe that Frege would agree.

The propositional calculus is an abstraction from the predicate or functional calculus. I think this because I believe that there are no sentences or propositions that, under analysis, are true or false and that lack predicative form. True sentences or propositions always involve predication. Of course there can be one-word sentences, and the one-word sentences may not be predicates. But insofar as they have truth value, it seems to me that the single words mask an implicit structure, involving some sort of implicit predicational attribution. I cannot make sense of having truth value apart from predicational attribution. The propositional calculus treats only logical operations on sentences or propositions. So predicate structure is suppressed or abstracted from. This correction seems to me not to affect significantly any argument in the paper.

The second correction has to do with the notions of logical validity and logical consequence. It requires more comment.

On p. 106 [CHECK–this is of the original] I include validity among modal notions. On p.

110 [CHECK] I write of Frege's conception of logical consequence as being in terms of necessary truth. These remarks were mistaken. The mistake was in effect corrected in "Frege on Apriority" p. 68 [CHECK--this is of the original], where I indicate that Frege understands the validity of logical laws in terms of the generality of their ultimate ground--the ground (such as fundamental logical axioms and inference rules) which justifies them--, not in terms of necessity. In fact, Frege seems to understand a truth's necessity in terms of the generality of its ultimate ground. He does not seem to treat it as an autonomous notion.

This point is of considerable historical and conceptual importance.<sup>1</sup> Relations among epistemic notions, like apriority, modal notions, like necessity, and formal notions, like generality and structure, have a long history in logical theory. Often the three types of notions were run together. Aristotle's logical theory emphasizes modality over form in understanding logical truth and good deductive inference ("following from"). In the middle ages, there emerged independent intuitive conceptions of logical validity and logical consequence that centered on form and structure rather than modality. Nearly everyone in this tradition took logical truths and logical consequences to be necessary. But the intuitive conceptions were not themselves modal conceptions. The oldest versions of such conceptions explicated logical truth and logical consequence in terms of containment relations among structures in the world that were reflected by logical relations among logical forms of proposition or argument .

These intuitive logical notions eventually received theoretical explication in terms of model- theoretic notions of validity and logical consequence. The intuitive notions, however, are distinct. They are notions of truth and truth preservation-- not notions of truth, or preservation of truth, in a model. The intuitive notions are also mathematically inspecific (unlike the model theoretic notions) and allow for a variety of conceptions of form, structure, and generality.

The key feature of the intuitive notions of logical validity and logical consequence is that they explicate logical truth and deductive truth preservation in terms of logical form and logical structure. They also take logical truths and logical consequence to exhibit a strong form of generality. Usually logical structure is understood to be an aspect of “the world” or of a subject matter. The generality intuition plays a role in determining what counts as logic, or as a logical constant.

The mistake that I made in “Frege on Truth” is that I did not recognize the importance of sharply separating this tradition from modal conceptions of logical consequence. It is not that logical consequences are not necessary or that there is anything wrong with modality. It is that the notion of logical consequence can be conceived without appeal to it—in terms of notions of formality, structure, and generality. I believe that this has clearly become the dominant conception in contemporary logic.

Frege is an important representative of the tradition that understands logical truth and good deductive inference in these terms. The near absence of modality in Frege’s conception of good deductive inference, and the prominence of these other notions (formality, structure, and generality) is extremely striking and of great historical importance. In fact, I think he made the key contributions to this tradition that enabled Skolem, Gödel, Tarski, and others to provide the theoretical understanding of the intuitive formal notions of logical truth and logical consequence, and of logic and deductive inference, that flowered in model theory. Frege’s non-modal conception of the key logical notions, including his (largely) non-modal explication of rules of inference in terms of assignments of truth values, his breakthrough work on logical form, his functional explication of the truth of whole sentences (or thoughts) in terms of the denotations of the parts of sentences, and his firm association of logical truth with the generality of the ultimate

bases for its justification and the generality (in a different sense) of its application to subject matters—are all fundamental and distinctive components to the intuitive conceptions. In fact, Frege brought these components to a level of formal sophistication that was unprecedented. His elaboration of them formed the basis for subsequent, modern theoretical conceptions of the intuitive notions.

Frege's grip on the pre-theoretical notions of logical truth as logical validity and of good deductive inference as formal logical consequence is deeper than that of anyone prior to those who systematized the intuitive notions in model theory.

Frege's philosophical orientation is quite different from those of most of his successors. Like all others who had the intuitive notions of logical validity and logical consequence who preceded the development of model theory, Frege worked only with the notion of truth. He did not connect that notion with a notion of truth in a model (which itself is, strictly speaking, not a notion of truth). Whereas domain variation has become a basic tool of model theory, Frege evaluated logical truth entirely by reference to the (actual) world.

This difference is easily overrated. Frege is, in this respect, like every other representative of the intuitive tradition prior to the emergence of model theory. Even most of the early expositions of model theory, including Tarski's original model-theoretic explication of logical consequence, do not employ domains or domain variation.<sup>2</sup> Frege took the axioms of logic to carry ontological commitment to an infinity of objects (for example, the numbers). He is like Russell in this respect. He is unlike most modern logicians, who take logical axioms to have at most commitment to the existence of one object. This difference in logical conception is compatible with his sharing the non-modal, formal conception of logical truth and logical consequence.

Frege had a different conception of the role of truth and of semantics in logical theory than his successors. He regarded at least certain attributions of truth as being in the object-language and as adding nothing to the sense of asserted non-semantical sentences. Frege did not regard the predicative attributions in these occurrences as being executed via a semantical predicate. Rather he understood sentences like " $2+2 = 4$ " as having the form, " $2$  plus  $2$ 's being  $4$  is the true", where "is the true" is a simple non-semantical predicate, formalized by the horizontal, that maps the truth value of " $2$  plus  $2$ 's being  $4$ " onto the truth value truth (given that the truth value of the nominalization is truth). It maps the denotations of nominalizations that denote the false onto the truth-value falsehood. And it maps the denotation of every other singular expression onto the truth value falsehood.

Further, Frege regarded " $2 + 2 = 4$ " as having the same sense as "it is true that  $2+2 = 4$ ". Presumably he regarded the sense of the nominalization of " $2+2 = 4$ " as having the same sense as that if the result of applying his predicate "is the true" to the nominalization. And he clearly regarded "it is true that  $2+2 = 4$ " as having the same sense as " $2+2 = 4$ ".

This redundancy account of the sense of the predicate "it is true that", which he parsed as "is the true", is surely false. In fact, the sense and denotation of the predicate "is the true" (the horizontal), in these uses, is wrapped up with Frege's commitment to extensions of concepts. The predicate denotes a function from truth values to truth values—where truth values are extensions. Since his notion of an extension of a concept is defective, his non-semantical notion of truth, expressed by the horizontal, is doubly defective.

For all that, Frege did have a semantical predicate that can be employed so as to fit Tarski's truth schema. This is the predicate "denotes" (bedeutet) applied to the relation between sentences and the truth value truth. Frege regarded "denotes" is a two-place relational predicate.

Frege frequently uses this predicate in doing some of his deepest and most influential work. Frege's use of this predicate, and his explication of relations between sub-propositional logical structure and propositional logical structure in terms of it, grounds my attribution to him of the intuitive notions of logical truth and logical consequence. These are intuitive notions of truth and truth preservation, respectively, explicable in terms of the logical form of sentences and arguments, on one hand, and logical structure to which such sentences are semantically related, on the other.

Let me turn to the third of the three things I want to do in this postscript.

Thomas Ricketts holds that Frege “has no semantic conception of logical consequence in the post-Tarskian sense of ‘semantic’”, and “lacks any general conception of logical consequence, any overarching conception of logic.”<sup>3</sup> He does not specify what he thinks the post-Tarskian sense of ‘semantic’ is.<sup>4</sup> It is obvious that Frege did not present a full-blown model theory.<sup>5</sup> Ricketts seems, however, to be claiming that Frege did not have the intuitive notions of formal logical validity or logical consequence, or any other general intuitive conceptions of logic. I want to evaluate some of these claims. I think that they are not well grounded.

One reason Ricketts gives for his claims is that Frege never states a “defining criterion of the logical”.<sup>6</sup> This seems to me not a strong reason. The fact that Frege did not hazard a defining criterion for logic is hardly a ground for holding that he lacked a conception of logic or logical consequence. Frege's use of his two key notions of generality (quantificational generality in the grounds for a truth and generality of application to all subject matters in ontology of logic) together with his practice of explaining formal deductive reasoning exhibits a deep conception of logic, pushed through with greater consistency than any predecessor did. These key notions and Frege use of them do not constitute a criterion for logic or even conception of logical

consequence. Frege's theoretical development of formal logic conjoined with his deep insights into the semantical structures associated with logical form are what shows that he has the intuitive formal conception of logical consequence. He did not need to give a definition or criterion.

Ricketts also claims, "Frege is not committed to the claim that every truth couched in topic-universal vocabulary is a logical law."<sup>7</sup> I think that we do not know whether Frege was committed to this claim. He does not explicitly make the claim, to my knowledge. But the claim is not one that every proponent of the intuitive conception of logical consequence would accept. The truth of the claim is not to be taken for granted in any case. For example, in first-order logic  $(\exists x)(\exists y)(-x = y)$  is not a logical law or even a logical truth, even though it is true and is couched in topic-universal vocabulary. I believe that Frege would have regarded this proposition as a logical truth, but only in virtue of his claim, which he knew to be controversial, that arithmetic is logic and that numbers are logical objects. There are, I believe, other truths (either the continuum hypothesis or its negation) that can be couched in the topic-universal vocabulary of second-order logic but which most logicians do not regard as logical truths. What Frege would have thought about such cases, if he had considered them, is hard to say.

What I want to bring out is that the claim that Ricketts cites, though it might have been attractive to Frege, was actually at least as controversial as particular aspects of Frege's logic. Frege had doubts about these aspects himself.<sup>8</sup> Even if Frege accepted the claim, it would have been bad dialectical strategy for him to assert a criterion for logic or logical truth that was at least as controversial as the most obviously controversial parts of his own logic—the parts on which his logicism about arithmetic hinged.

Much of Ricketts' case for holding that Frege lacked the concept of logical consequence seems to lie in claims that he lacked a truth-conditional semantics—or indeed any semantics

relevant to understanding logical notions.<sup>9</sup> Ricketts' claims are associated with the aim of trying to understand Frege's type-restrictions and his conception of truth. But the claim that Frege lacked a truth-conditional semantics is, I think, deeply and obviously misguided. Where does such a claim come from? I believe that there are no good textual grounds for this claim. I will discuss the purported bases for it.

One purported ground is a reading of Frege's argument against the definability of truth. Frege's argument goes as follows: "...in a definition [of truth] certain characteristics [such as corresponds to reality] would have to be specified. And in application to any particular case the question would always arise whether it were true that the characteristics were present. So we should be going round in a circle. So it seems likely that the content of the word "true" is sui generis and indefinable."

As it stands, Frege's argument is an enthymeme. Frege does not explain why a definition of truth and meta-level propositional attitudes must be relied upon in order to decide whether a proposition is true. One might simply make non-meta-level judgments and then infer the truth of the judged contents from them. It is unclear to me whether Frege's views are vulnerable to this reply since his views are not explicit.

One might try to save Frege's argument, however, by supplementing it with three further premises. The first is that ordinary non-factive judgments presuppose the notion of truth in some looser way than actually employing or containing the notion. Frege would hold that non-factive judgments at least aim at truth and that this aim constitutes a presupposition of any application of belief or non-factive judgment. I believe that this first premise is certainly true. The second is that a definition gives an epistemic order of priority or justification: the defined term is known to apply through applying the definition. Although few people nowadays understand definitions in



the formal sciences in this way, Frege did. Third if a definition, understood in the way indicated in the second premise, defines a notion that is presupposed in a full account of what it is to apply the definition, the definition is inadmissible. Even if one tried to substitute the definiens for the definition in the presupposition, the definition, understood in the light of the second premise, would be involved in epistemic circularity. I believe that Frege probably accepted all of these premises. I believe that this reconstruction is a fairly plausible conjecture about how Frege was reasoning.<sup>10</sup>

Ricketts reads the argument as showing not only that truth is indefinable (its only explicit conclusion) but that truth is not a property. He wants to infer from this result that Frege lacked a truth-conditional semantics. I will later question this latter inference. The inference is the key issue. But I want to begin by discussing Ricketts' reading of Frege's argument against the indefinability of truth.

I find quite unpersuasive the reading of the argument that takes it to be directed against taking truth to be a property. There is nothing in the text of the argument to support this reading. In Ricketts' exposition the claim depends on the idea that to take truth to be a property is to hold that to recognize a thought to be true must be to recognize a mental representation to have a particular property.<sup>11</sup> But this is just the mistake, alluded to in the previous paragraph, of conflating object-level judgments with judgments that a proposition is true. No opponent need accept this view.<sup>12</sup> I see no decisive reason for thinking that Frege made the mistake of attributing such a view to his opponents. Certainly an opponent need not make this mistake.

Frege does not draw from the argument against the definability of truth the conclusion that Ricketts attributes to him. Frege never says that the argument shows that truth is not a property. Frege's argument is at best obscure. I believe that Ricketts' reading is not only unsupported by

the text; it is uncharitable to Frege. The only charitable reconstruction of it that I know of (given above) does not support such a conclusion. Frege clearly did not give the argument against the definability of truth to show that truth is not a property.

Still, Frege did believe that truth is not a property, at least not a property of thoughts or sentences. More cautiously put: he was uncomfortable with this way of construing truth.<sup>13</sup> Although property is not a theoretical term that Frege uses in his theory, his overall theory is naturally read as not conceiving truth as a property.

Frege explicitly makes use of four expressions that are conceptually associated with truth. There is the horizontal–translated “is the true”–which denotes a function from truth values to truth values.<sup>14</sup> There is, second, the singular expression “the true”, which denotes the extension of the concept or function denoted by the horizontal. Third, there is “denotes” applied to sentences. This expression denotes a function from sentences, or their nominalizations, to truth values. Fourth, there is the predicate “determines” (or in Church’s usage “is a concept of”) as applied to thoughts and components of thoughts. This expression denotes a function from thoughts to truth values (more generally thoughts and thought components, or senses, to the denotations of expressions that express them).

Denotes the truth-value truth could perhaps be seen as a property of sentences. There would be a corresponding property of thoughts, determines the truth-value truth. But Frege clearly thinks that the fundamental notions are denotation (determination) and the truth-value, truth. The truth-value truth, which judgment “moves toward”, is an object, not a property. “Denotes” is a two-place relational predicates under which sentences and truth values, pairwise, fall. Analogously, for “determines”.<sup>15</sup> Even allowing for Frege’s loose, non-technical use of “property”, none of these expressions applies to or denotes properties of thoughts. It seems to

me that this point about Frege is obvious. One does not need a strained interpretation, lacking textual evidence, of the argument against the definability of truth to support it.

It is clear that Frege thought that the sense of the horizontal function sign does not add to the sense of declarative sentences to which it is attached. He seems to make a similar claim for the two-place predicate “denotes” as applied to a sentence and the true and for the analogous two-place predicate that is applied to a thought and the true. He seems to regard all three predicates (or predicate applications) as having this sense-redundancy feature.<sup>16</sup>

Ricketts makes much of Frege’s claims that truth is a goal of judgment, not a property of thoughts, and of Frege’s further claim that “it is true that-----” has the same sense as “-----”, for sentential fillings of the blank. Neither of these claims seems to me a high point in Frege’s work. As I noted, the latter claim is surely false. There is no question that Frege made them, however. Ricketts draws strong conclusions from these claims that are intended to show that Frege could not have had a semantics—and consequently could not have had intuitive semantical notions like logical validity and logical consequence. I believe that here again the conclusions are not supported by textual evidence.

One conclusion is that although, according to the redundancy view “true “ has a sense that does not add to the sense of sentences in “it is true that-----”, “true” does not “mean” (denote) anything.<sup>17</sup> I can find no evidence for this view. Ricketts does not present any. Frege clearly explains what concept the horizontal denotes. There are other passages in which Frege speaks of the denotation of “true”, without indicating that he does not mean what he says.<sup>18</sup>

Another of Ricketts’ conclusions is that the relation between a thought and a truth value is “not describable in a sentence”.<sup>19</sup> This conclusion does not follow from either the redundancy view of the sense of “it is true that” or from Frege’s views that truth is the goal for judgment and

that truth is not a property of thoughts. Again, Ricketts does not provide the slightest textual support for attributing the conclusion to Frege. Frege quite explicitly describes the relation between a thought and its truth value: "...there must be something associated with a sentence which is different from the thought, something to which it is essential that the parts of the sentence should have denotations. This is to be called the denotation of the sentence."<sup>20</sup> The denotation's being associated with the thought (by being a denotation of the sentence) is the described relation between thought and truth value. Frege describes it in these two sentences. Clearly he could have described it in one! I know of no reason to think that Frege thought that the relation could not be described in a sentence in a formalized language—formalized within the Begriffsschrift. Moreover, it can be.<sup>21</sup>

It is well-known that Frege claims that logic itself would be dispensable if our language were a perfect language. Logic could be read off the language without any need to formulate it. Since our language is not a perfect language, he indicates that we have a need for logic. He also implies that the word "true" is indispensable in explaining logic, and even in "laying the foundations" of logic.<sup>22</sup> It should be noticed that Frege explicitly states in this passage that "true" is indispensable in logic. Its uses in logic would be dispensable only in a perfect language that had no need for a logic that abstracted out logical laws and inference rules.

It is an open question just which term "true" Frege is claiming would be dispensable, along with logic, if our language were perfect. It is likely that he has in mind the semantical predicates "denotes" as applied to sentences and "determines" as applied to thoughts. "Is the true", the horizontal, is a primitive in Frege's Begriffsschrift.

The conclusion that the relation between a thought and a truth value is "not describable in a sentence" might be encouraged by imagining a perfect language in which both logic and

semantical occurrences of “true” are dispensable and then reflecting on Frege’s redundancy theory of the sense of “true”. Frege appears to hold the redundancy-of-sense theory about all three “truth predicates” in his system. One might reason as follows: ““-----“ denotes the true” has the same sense as “-----“. The predicate “denotes” would not be needed in the perfect language. So there is no sentence describing the relation between sentences and the truth value, true, in a perfect language. Analogous reasoning could be extended to any analog of “denotes” that applies to the relation between thoughts and the truth-value, true.<sup>23</sup>

This reasoning would be unsound.

In the first place, Frege never says that “true” is dispensable in a perfect language—only that logic is dispensable in a perfect language and truth is indispensable for logic. He does not claim that in a perfect language there would be no non-logical (special-science) uses for “denotes” or its analog, “determines”. So the second step is doubtful.

In the second place, since sentences “-----“ and ““-----“ denotes the true” have, according to Frege, the same sense, the former sentence specifies the relation just as well as the latter. So the move from the second step to the conclusion is fallacious. I know of no reason to think that Frege would deny that in a perfect language either the denotation of “the true” or the relation between thought and the true would be denoted, or made reference to. I will return to this point. There are several instances in Frege’s work where he says that sentences with apparently disparate ontological commitments have the same sense. His view is that adequate understanding of the senses of the sentences enables one to think about each sentence’s commitment equally well through thinking a thought through the other.<sup>24</sup>

In the third place, Frege explicitly calls truth a predicate and indicates that it applies to a relation between a thought and its truth value (when it is true). As I have indicated,

he often describes the very relation that is supposed to be “not describable”.<sup>25</sup> So the conclusion, in the third step of the argument, is surely not Frege’s conclusion.

One further conclusion that Ricketts draws is that “there can be, in a sense, no genuine theorizing about logic. There is only theorizing within logic—the proof of derived logical laws from basic logical laws and the application of logic in formal proofs within the framework of the *Begriffsschrift* to the laws and facts uncovered by the special sciences.”<sup>26</sup> This conclusion seems to be associated with the claim that “Frege’s understanding of truth, in precluding a genuine truth-predicate—one usable in generalizations—also rules out truth-conditional semantics, rules out, that is, a theory of how (the thoughts expressed by) sentences are determined to be true or false by the items referred to in them.” Ricketts adds that the attempt to state truth conditions of the sentence “Sea water is salty” “yields, on Frege’s view, only the tautology that sea water is salty if and only if sea water is salty.”<sup>27</sup>

It would have been well to apply *modus tollens* to this form of reasoning and find what is wrong with it rather than to accept the patently unacceptable conclusion that Frege’s view of truth is in such clear contradiction with his development of a truth-conditional semantics.

Frege’s semantics was not intended to justify the object-level or ground-level logic proper. It was intended to yield deeper understanding of it.<sup>28</sup> The *Begriffsschrift* was intended as a language that exhibits the fundamental justificatory relations between basic logical truths and arithmetical truths. Since the semantical elaborations are not justificatory, it is not surprising that Frege sees them as not part of a perfect language suitable for expressing basic logical truths and derivations from them that issue in arithmetical theorems.

Ricketts seems to be appealing to Frege’s redundancy theory of the sense of “true” to support his remarks about truth conditions being tautologies. “Tautology” is Ricketts’ term.

Used in the usual modern way, I think that using it to characterize Frege's thinking about such statements is anachronistic. The application also betrays a misunderstanding of Frege's conception of sense.

I have noted that Frege regarded applications of the semantical predicate "denotes" to sentences and truth values as having the same sense as the relevant sentences. He also applied his sense-redundancy theory to the analog of "denotes" ("determines") that applies to the relation between thoughts and truth values. Ricketts infers that Frege's sense-redundancy views commit him to holding that

"sea water is salty" denotes the true if and only if sea water is salty  
has the same sense as

sea water is salty if and only if sea water is salty.

Or to take account of the fact that for Frege truth applies more properly for thoughts than for sentences, he would be committed to holding that the two sides of the biconditional in

the thought that sea water is salty determines the true if and only if seawater is salty  
express the same sense.

I believe that up to this point Ricketts' reasoning is correct. It does not follow, however, that semantics is empty of scientific value in Frege's view. Before his acknowledgment of Russell's paradox, Frege regarded the two sides of the biconditional that constitutes his naive comprehension principle, a simple corollary of Basic Law V, as expressing the same sense.<sup>29</sup> He hardly regarded this corollary as a tautology, in any sense that would count it empty or without scientific value, or as not part of "genuine" theorizing within logic.

The two sides of the truth-conditional biconditional, like the two sides of this naive comprehension principle, have the same cognitive value, according to Frege. How is it that they

can be of scientific value?

At the beginning of “On Sense and Denotation”, Frege contrasts statements of the form “ $a = a$ ” with statements of the form “ $a = b$ ”. He says of the former that they hold apriori. He says that the latter “often contain very valuable extensions of our knowledge”.<sup>30</sup> This may suggest that statements of the form “ $a = a$ ” do not contain valuable extensions of our knowledge. But Frege never says that they are lack scientific value or usefulness. He simply says that statements of the two forms have different cognitive values and that statements of the “ $a = a$ ” form are apriori.

I believe that especially in cases where the sense of an expression has logical structure, Frege thought that there is room for informativeness or scientific value in expressions of the same thoughts in different ways. Sense is cognitive value. Cognitive value is what a fully rational, fully informed thinker, with complete mastery of the language, grasps.<sup>31</sup> Only a fully informed individual with complete logical understanding would find the semantical biconditionals or the corollary of Basic Law V (imagining that it had turned out to be a logical law) to be cognitively uninformative. Where there is incomplete understanding, expression of the same thoughts in different ways might well be of cognitive value. Frege makes it clear that he thinks that logic is necessary only because we have an imperfect language. The same point would apply to the biconditional theorems in a semantics for logic.<sup>32</sup> This imperfect condition of ours is both cause and effect of incomplete understanding.

Frege develops this sort of point in his account of definitions. Definitions cannot add to knowledge in the most idealized sense of “add”. The definiens has the same sense as the definiendum. If one has a perfect grip on the structure of a thought, then the definitions are dispensable in proof, and they are always, logically speaking, dispensable. But even definitions can be theoretically fruitful and of scientific value—especially if they express structure in a



perspicuous way. For one's grip on the structure of a thought goes through language, and often a definition is needed by the reasoner to hold in mind the complex sense structure associated with a word, or to make clear what had been only imperfectly or incompletely grasped.<sup>33</sup>

The fact that Frege regarded his naive comprehension principle as expressing the same senses on each side of its biconditional, and the fact that he regarded semantical truth-theoretic biconditionals in the same way, shows that he believed that more logical structure is involved in a sense or thought than is uncovered even by the linguistic structure of any one sentence in his logical notation. Thus the thought expressed by a quantified sentence which in Peano notation would be  $(x)(Fx \leftrightarrow Gx)$ , is committed not only to first-level functions denoted by "Fx" and "Gx", but to courses of values associated with those functions. An ideography truly adequate to the thoughts being expressed would have to have an extra-dimension, to account both for inferences associated with quantification into the function's argument place and for inferences associated with the identity of the courses of values.

Frege's remarks in his late "My Basic Logical Insights" (1915) take on a deeper significance in the light of these points. Frege writes, "If we had a logically perfect language, we would perhaps further need no logic or we could read it [logic] off the language. But we are at a vast distance from being in this condition" [my emphasis].<sup>34</sup> It appears that at least during the time when he was proposing Basic Law V, even his own ideography was not seen as one in which one could read off all relevant thought structure from any given sentence. Inferences from a thought involving extensions of concepts are not explicit in a sentence of simple quantification or predication. But both structures are determined in the sense or the thought. So more structure resides in a sense or thought than is uncovered by the linguistic structure of any given sentence even in the Begriffsschrift. A person who fully and deeply understands the right side of these

biconditionals would fully and deeply understand the relational structure indicated in the left sides.

I think that the same point applies to Frege's understanding of the science of semantics. He thought that the analogs of Tarski's biconditions do express the same sense on each side of the bi-conditional. A person who fully understands the thoughts expressed by the bi-conditionals would learn nothing from them. In a perfect language, both structures would be mentioned. Frege regards logic as worthwhile given that languages are not perfect (do not in their form reveal all relevant logical inferences from the thoughts they express). He would, or certainly could, take the same view about semantics. It is scientifically worthwhile in explaining the foundations of logic, given that our languages are not perfect and given that incomplete understanding of thoughts expressed in our languages is common among users of the language—including users of logic.

It is important to recognize differences between Frege and his successors in logical theory. It is also important to see Frege's work in the broad perspective of the history of logic. Frege clearly initiated modern semantics. He had an unprecedented grasp of logical form and logical structure. He showed how the semantical structure associated with sentential parts (or thought components) are systematically associated with the truth value of sentences or thoughts. His motive in producing this account lay in accounting for good deductive inference. He gave no central explanatory role to modality in his account. He certainly had and deepened the understanding of the intuitive notion of (formal) logical consequence. His conception of logic centered on the generality of its axioms and its subject-matter applications.

While it is certainly true that he did not give a model theory, did not raise the questions that led to the flowering of meta-logic (most notably in the completeness and incompleteness theorems), did not foresee the seriousness of the semantical paradoxes, and may have thought of

semantics as more loosely related to logic than it in fact is, his conception of formal consequence, understood semantically, engendered the explosion of progress in semantics and meta-mathematics. His conception is in fundamental ways continuous with that progress.

Frege also differed from many of his successors in having superior insight into the ontological entanglements of logic and superior insight into its epistemology. He had a forthright conception of the dependence of the truths of logic on a subject matter. Logic for him is the most general science of being. This view is not less true in light of his type-theoretic understanding of being. In his view of logic as a science of being, he was joined by Russell and Gödel but by few others among his great successors. I believe that on this matter he had deeper insight than Wittgenstein, Carnap, Tarski, and even Quine. He also saw clearly that logical knowledge lies in understanding—fundamentally the kind of understanding that goes with competence in making good inferences and understanding logical truths. Little or no further warrant is to be gained through semantical work applied to logic. Such work clarifies and deepens understanding without fundamentally strengthening warrant.

Let me turn now to the fourth thing that I want to do in this postscript. I want to speculate a little on Frege's views about semantics in a somewhat wider context. As I have noted, late in his career Frege states that "true", presumably as formalized by the semantical predicates "denotes" and "determines", is indispensable in logic, but that logic is dispensable in a perfect language. I have noted that this view does not imply that the semantical predicates are dispensable in a perfect language. Frege uses his semantical predicates in expounding his inference rules in his "extensional" logic (his logic that does not explicitly deal with senses). He would also have used the semantical predicates (at least "determines") if he had elaborated the logic of oblique contexts—contexts of propositional-attitude attribution—that do explicitly quantify

over senses.

I raise two questions. First, if Frege had elaborated a logic for oblique contexts, would he have maintained that a logic for such a language is dispensable? Or equivalently, did Frege intend to include a logic of propositional-attitude attribution when he claimed that logic is dispensable in a perfect language? Second, given that “logic” and presumably a semantics for logic are dispensable in a perfect language, are there non-logical uses for the semantical predicates that Frege would have admitted?

I think that we do not know the answer to the first question. There is some reason, however, to believe that he may have regarded the semantical predicates as having a different role in a logic that explicitly deals with senses. There is some reason to believe that Frege would have thought that some thoughts (thought contents) like the numbers are implicitly present in every subject matter. Hence they would be a proper subject for the universal science of logic. Frege’s Platonism implies that thoughts that are logical truths (including arithmetical thoughts) are in a sense implicit in any discourse, no matter what the topic. What this means is no matter what subject matter a piece of language is used to discuss, that piece of language contains expressions that are related (by the expression relation) to logical thoughts. In this sense, logical thoughts are just as ubiquitously related to any discourse as numbers or truth-values. Of course, unlike numbers and truth-values, they are not part of the (denoted or determined) subject matter in the discourse of a language formulated in extensional logic, one that formalizes no oblique contexts from natural language.

But suppose that one were to see logic as including formalizations of oblique contexts. One could introduce an impersonal conception of assertion, idealized to apply to the theorems of logic—it is to be rationally and ideally judged that \_\_\_\_ (where the blank is to be filled by axioms

or theorems of logic). In an “intensional” logic that formalized such sentences, thoughts would become part of the denoted subject matter of the language. They would not be implicit in a subject matter merely by being expressed. Since such a subject matter would, from Frege’s Platonic point of view, be just as relevant to and present any subject matter as the truth-values and the numbers.

If this speculation were in accord with Frege’s conceptions, as it seems to be, then the most general principles governing relations between thoughts and truth values might well be something for a general science of being to characterize explicitly. I think that it is quite possible that Frege had only the narrow notion of logic in mind—logic not including a logic of that formalizes discourse about thoughts or senses—when he stated that logic is dispensable in a perfect language. From such a perspective, the broader logic would include an account of the relation between thoughts and their truth-values in its object language. Of course, we now know that things cannot be so simple. But Frege was not very sensitive to the threat of semantical paradox, and related paradoxes.

I think that we do know the answer to the second question—the question whether Frege would have admitted special-science uses for the semantical predicates. There is every reason to think that he would have acknowledged that semantical predicates can be incorporated into a special science. They can be incorporated into a science that deals with the relations between sentences and truth, and with relations between truth and thought contents, including non-logical, empirical contents. Such a science could deal with thought contents that are clearly not part of every subject matter, even implicitly. Such a science can be formalized within the Begriffsschrift.

## Footnotes

1. These historical and conceptual points are developed at much greater length in my “Logic and Analyticity”, Grazer Philosophische Studien 66 (2003), pp. CHECK. There is a discussion there, especially in footnote 37, of some of the points about Frege made below.

2. Alfred Tarski, “On the Concept of Logical Consequence” (1936) in Logic, Semantics, Metamathematics, (Indianapolis, Hackett, 1983). Cf. Tim Bays, “Tarski on Models” The Journal of Symbolic Logic 66 (2001), pp. 17001-1726.

3. Cf. e.g. Ricketts’ “Logic and Truth in Frege” Proceedings of the Aristotelian Society, Suppl. vol. 70, 1996, p. 124 . In other respects, I like the seriousness of Ricketts’ sensitivity to respects in which Frege’s work is foreign to some elements in the current philosophical climate. But I think that some of the main points of his Frege interpretation are very far off base and strikingly lacking in specific textual support.

4. To be maximally specific: I take the intuitive notion of logical validity to be truth grounded or correctly explicable (in any of a number of different ways) in terms of logical form and logical structure. The intuitive notion of logical consequence is preservation of truth grounded or correctly explicable in terms of logical form and logical structure. Logical form is a feature of sentences or propositions. Logical structure is usually taken to consist of objects, properties, relations functions, sets “in the world”. The ontology varies with the conception. Grounding or explication can be understood in various ways. In fact, “logical” can be understood in various ways that are compatible with the intuitive notions. The most common conception of logicity (which Frege represents) involves reliance on the generality of application of the logical constants.

I take “semantical” to apply to relations between signs, or (or in a broader usage of “semantical”) thought components, and entities that they mean, refer to, express, or the like. Semantics need not be model-theoretic.

5. It seems to me questionable, however, to claim even that Frege lacked an approximation to the concept of truth in a model. Although he does not give a systematic model theory, and although he does not employ the modern notion of a set (in terms of which models are explained), his reasoning in sections 10 and 31 of The Basic Laws of Arithmetic suggests a grip on the theoretical forms of reasoning that came to constitute model theory and the early model-theoretic explications of logical consequence.

6. “Logic and Truth in Frege”, op. cit., p. 124.

7. Ibid.

8. Cf. my “Frege on Extensions of Concepts: From 1884 to 1903” for the point about Frege’s doubts. Cf. also “Logic and Analyticity”, op. cit., note 23, and Appendix I (“Logic: First-order?”

Second-order?") of that paper. In the latter paper, I discuss why on natural conceptions of logic and intuitive logical consequence (or logical validity—for sentences or propositions), one should not accept the claim that Ricketts takes as criterial for having the intuitive notion of logical consequence. The natural conception of logic that I develop there is not Frege's. The point is that the intuitive notion of logical consequence is not constitutively committed to the claim. The claim would have been attractive to Frege, but whether he is committed to it seems to me unclear. His not explicitly making the claim can be explained by either substantive or dialectical reasons.

9. Ricketts claims that "much of what we tend to think of as Frege's semantics—is not storable within the framework of the *Begriffsschrift*. Frege's universalist conception of logic gives it an anomalous status", "Logic and Truth in Frege", *op. cit.*, p. 128. This claim is quite inspecific. Ricketts may just be pointing out that some of Frege's remarks about functions in German or English would be subject to "the-concept-horse" problem. This point would be correct but would have no deep implication for the role of semantics in Frege's understanding of logic. Neither Frege's logic nor his semantics need treat functional expressions as denoting what can be denoted by singular expressions (although I believe that there is no good reason why a functional expressions and singular expressions cannot bear different denotation relations to the same entity, as long as type-restrictions are maintained as to what sort of function is being denoted). Ricketts seems to mean that no semantics that Frege had can be formulated in the framework of the *Begriffsschrift*. Again, this claim would be inspecific. If the point is merely that semantical terms (like "bedeutet") are not among the logical constants that Frege employs, that is clearly true. This point certainly could not by itself be used to show that Frege did not have a semantical conception of logical consequence or logical truth. Many who have the intuitive semantical conception, logical consequence, would be happy to agree that semantical notions are in a sense not logical constants proper—components of the axioms or even inference rules of logic. If the point is that even taking Frege's semantical notions as non-logical constants, one cannot formulate Frege's semantics in *Begriffsschrift*, this seems to me quite incorrect. Ricketts does not give any argument for the claim. I see no reason to think that the semantics for Frege's first-order functional expressions cannot be formulated in a semantics within the framework of a higher-order logic that distinguishes function-denotation from object-denotation. Function-denotation for first-level functions is a two-level relation between an object (a sign) and a first-level function. One simply needs to employ different semantical expressions for the different types of expression being explicated (signs for objects, signs for first-level functions, signs for second level functions, and so on). In principle, the type theory continues without limit, and the semantics can do so as well.

It is, of course, true that signs are special objects—the province of a special science. But in various places Frege makes it clear that he thinks that the fundamental representations are thought contents and thought-content components. I see no deep reason why Frege could not have regarded semantical predicates as logical constants. If he had developed an logic for attributions of propositional attitudes, I think that he may well have done so. (I discuss this point below in the text.) I see no deep reason why he could not have regarded thoughts as logical objects, as long as the thoughts themselves contain no non-logical components.

Of course, no type theory of the sort that Frege proposed has a single notion of generality or universality that applies to "everything". Frege clearly envisioned an intensional logic, of the

sort that Church developed. (Cf. Frege to Russell, 12/28/1902.) It is far from obvious that he thought that thoughts, in particular those that contain purely logical vocabulary and apply to logical objects and functions, are objects of a special science. They appear to be as universal, for him, as the numbers.

10. For Frege's argument against the definability of truth, see "The Thought", in Collected Works (Oxford, Basil Blackwell, 1984), p. 353; p. 60 in original; and "Logik" (1897), in Nachgelassene Schriften, Hermes, Kambartel, Kaulbach, eds. (Hamburg, Felix Meiner Verlag, 2<sup>nd</sup> edition, 1983), p. 139-140; also in Posthumous Writings, Hermes, Kambartel, Kaulbach, eds. Long, White, trans. (Chicago, University of Chicago Press, 1979), pp. 128-129. Ricketts claims that in the argument Frege relies on his elucidation of judgment as recognition of truth ("Truth and Logic in Frege", op. cit. p. 129); he holds that Frege's notion of judgement applies only to truths and is thus "quasi-factive". He sees the argument as an attempt by Frege to show that the definability of truth would make judgment conceived in this factive way impossible. Ricketts does not note, however, that the argument never invokes the notion of judgment. The notions that Frege does use in the argument are manifestly non-factive ("decide whether a representation corresponds to actuality", "compare representation and reality"). It is possible that Frege is assuming some connection between factive and non-factive judgment, but he nowhere makes this explicit. So I regard Ricketts' reading as quite implausible. It does not accord with the text. I think that the argument is most charitably understood independently of any special notion of factive judgment, since Frege does not suggest any relation between the notions he does use and such a notion.

11. "Truth and Logic in Frege", op. cit., p. 131. I find Ricketts' account of the argument very obscure. But I understand it well enough to make the following points. I also think that the argument makes a charge of circularity not of regress.

12. I believe that Frege himself either makes this mistake or writes in a way that strongly suggests it. Cf. "The Thought", op. cit., p. 354; in Kleine Schriften, I. Angelelli, ed. (Hildesheim, Georg Olms, 1967), p. 345; p. 61 in original.

13. "The Thought", Collected Papers, op. cit. pp. 354-355; pp. 61-2 in original; Kleine Schriften, op. cit., p. 345.

14. Despite this translation, Frege clearly takes the horizontal to be more basic than the singular expression "the true" that denotes the truth value. Similarly, the concept denoted by the horizontal is more basic than the object which is the truth value. Explains the latter in each group in terms of the former.

As mentioned before, the horizontal maps anything other than a truth value onto falsehood.

15. Church called this the concept of relation. As with Frege's notion of denotation, the concept-of (or determination) relation between a thought and its truth value is a special case of the relation between any thought or thought component and what the corresponding linguistic expression



denotes. Cf. Alonzo Church, “The Logic of Sense and Denotation” in Structure, Method and Meaning: Essays in Honor of Henry M. Sheffer, P. Henley, H. M. Kallen, and S. K. Langer, eds. (New York, Liberal Arts Press, 1951). Frege is clearly aware of the possibility of using an expression for this relation. Cf. “Logic” (1897), in Posthumous Writings, op. cit., p. 229; in Nachgelassene Schriften, op. cit., pp. 140-141. If Frege had developed an intensional logic for attributions of belief, as Church did, I believe that he would have introduced the predicate “determines” into his formal scientific language.

16. To be clear: The three predicates are the horizontal, “denotes”, and “determines” (as applied to a relation between thoughts and truth values). Cf. “Logic in Mathematics” in Posthumous Writings, op. cit., p. 234; in Nachgelassene Schriften, op. cit., p. 252. He is quite explicit in counting these meta-level expressions predicates. He indicates that they apply to relations not to properties.

17. “Truth and Logic in Frege”, op. cit., p. 134.

18. “The Thought”, op. cit., p. 59 in the original. “Logic”

19. Op. cit., p. 135. Ricketts writes, “The relationship between a thought and its truth-value is not describable in a sentence—it is not a matter of a thought’s falling under a concept or of a relation’s holding between two objects. Rather, the relationship between the thought that Socrates is mortal and the True is linguistically expressed by an indication of the asserting force with which a sentence expressing the thought is uttered by someone who has recognized-the-truth of the thought.” I believe that in his exposition of this conclusion, Ricketts confuses the relation between the thought and the truth value with aiming at the truth value in judgment, which carries the force of assertion or judgment. This is exactly what Frege inveighs against—except that Frege is more concerned with the mistake of assimilating judgmental force to the relation between a thought and its truth value true than with the mistake of assimilating that relation to judgmental force. The relationship between a thought and its truth-value is indeed, and very explicitly so in Frege’s writing, a relation’s holding between two objects (a thought or sentence and a truth value). Frege nowhere states that the relationship between thought and truth value is not describable in a sentence.

There is a sense in which judgmental force, on Frege’s view, cannot be fully captured in any description or declarative sentence. Frege does make this claim. But he does not claim that the denotational relation cannot be described. As I have noted, he often describes the relation.

Rickett’s account is further hampered by a gloss of Frege’s word “einzigartig” (sui generis) as indicating that its content does not comfortably fit in any logical category (ibid, p. 132). No textual support for this gloss is given. What Frege says in the context of his use of the word does not support any such radical paraphrase.

20. Posthumous Writings “Introduction to Logic” (1906), op. cit., p. 194; in Nachgelassene Schriften, op. cit., p. 211.

21. There is no difficulty in principle in formulating a systematic type-theoretic semantics in the Fregean mode. A condition on such a semantics is that it be capable of proving formal analogs of

Sentence “Fa” denotes the true if and only if the denotation of “F” maps the denotation of “a” onto the true

and

“Fa” denotes the true if and only if Fa.

One might have semantical rules for functions (extensionally conceived) like:

(F)(“is a horse” f-1-denotes F if and only if (x)(Fx if and only if x is a horse)).

“f-1-denotes” in this usage is a two-level two-place concept, taking objects and first-level functions into truth values. “o-denotes” as applied to singular (object-level) expressions would be a single-level two-place concept, taking a pair of objects (a sign and an object) into truth values. Then the two “denotes” (presumably together with further semantical predicates for the quantifiers) would have to be coordinated in order to prove the analogs of the Tarski bi-conditionals mentioned above.

A semantical theory for n-order function signs would be given in a theory that is at least n+1 order. Frege himself thought that each function is equivalent to at most a function of level three. But this is probably not correct, on any natural construal, by Cantor’s theorem. (I am indebted to Terry Parsons on this point.) Although Frege may have hoped to collapse the type-theory into a three-level theory, it appears that, as with almost any type-theory, the semantics for a given level must always recede into a higher-level language. Still, there is no reason to think that on Fregean principles the semantical relations are not describable within the type-theoretic constraints.

22. “My Basic Logical Insights” (1915) Posthumous Writings, op. cit. p. 252; Nachgelassene Schriften, op. cit., p. 272. I believe that Frege had the notion expressed by the horizontal in mind in his discussion here. In “On Sense and Bedeutung” op. cit., p. 34 of the original, one can see the effects of his separating force from predication, in particular predication involved in the horizontal. He insists that force does not reside not in this predicate (as he has suggested in Begriffsschrift, Section 3). His sense-redundancy view is directed at the idea that saying a thought is true, or saying “it is true that -----“ captures the force of a judgment. In his formal work, he comes to regard “is the true” as the fundamental object-level predicate that applies to gerundized construals of sentences. But he also makes use of the meta-level predicate in various ways such as in his exposition of his axioms and in his statement of his inference rules. Cf. the last pages of section II of “Frege on Truth”.

23. Such an extension is strongly suggested by Frege’s remarks in “On Sense and Bedeutung”, p. 34 in the original.

24. One instance, associated with a corollary of Frege's Basic Law V is discussed in "Frege on Extensions of Concepts: From 1884 to 1903", section V of that article. Also see "On Concept and Object", p. 200 in the original; and "On Sense and Bedeutung", p. 34 of original: "These two objects [the truth values] are recognized, if only implicitly, by everybody who judges something to be true...". For further discussion of these issues, see "Frege on Truth".
25. Cf. "On Sense and Bedeutung", in the original, combine pp.27 and 33-34, and especially p. 34. Also, "Logic in Mathematics" (1914) in Post humous Writings, op. cit., p. 234; in Nachgelassene Schriften, op. cit., p. 252 (where Frege counts the meta-linguistic "denotes the true" a predicate); and "Introduction to Logic" (1906), Posthumous Writings, ibid, p. 194; in Nachgelassene Schriften, op. cit., p. 211. Frege's point in denying that truth is a property is partly to insure that predications of truth of sentences or thoughts specify a relation between the sentences or thoughts and the true. Of course, judgment and its force are yet further matters.
26. "Logic and Truth in Frege", op. cit., p. 136. Frege uses a semantical notion of truth in stating his inference rules. I believe that he believes that the truth predicate (cf. note 25) that expresses this notion, like the horizontal (which is non-semantical), is indispensable in the practice of logic. There is clearly a kind of generalization involved in these uses of the truth predicate, "denotes the true". Since I have discussed the role of inference rules in his system in some detail in "Frege on Knowing the Foundation", I will not enter into this issue further here.
27. "Logic and Truth in Frege", op. cit., p. 140. In the last part of his paper, Ricketts advances a deflationary interpretation of Frege's ontological Platonism appealing to various ordinary-language points about judgment. I believe that this part of his paper makes little contact with Frege's texts. Moreover, it does not consider the main passages where Frege gives his Platonism philosophical work to do—passages that I have discussed in "Frege on Knowing the Third Realm". So I will not be discussing these claims. I should add that I think that the deflationary surrogates are, from a purely philosophical point of view, thoroughly inadequate to the task of accounting for a number of philosophical considerations that Frege was aware of.
28. These points are discussed in "Frege on Knowing the Foundations".
29. Cf. note 24. And section V of "Frege on Extensions of Concepts: From 1884 to 1903".
30. "On Sense and Bedeutung", p. 25 of the original.
31. Cf. section VI of "Frege on Extensions of Concepts: From 1884 to 1903", section VI; "Frege on Sense and Linguistic Meaning", and the discussion of this point in the Introduction to these essays.
32. There is the additional familiar point that although the Tarski-type biconditional theorems in a semantics seem trivial, the derivation of them from the semantics for the parts of sentences (or thoughts) is not at all trivial. I believe that Frege was aware of the analog of this point for his own systematic semantics.

33. Frege, Foundations of Arithmetic, op. cit., section 70; “Foundations of Geometry II” in Collected Papers, op. cit., pp. 300-301, in the original pp. 302-303; “Logic in Mathematics” in Posthumous Writings, op. cit., p. 209, 216-222; Nachgelassene Schriften, op. cit., pp. 225-226, 234-240.

34. Posthumous Writings, op. cit., p. 252; Nachgelassene Schriften, op. cit., p. 272.