Essays on Frege
[Title Not Determined]

New Matter in the Frege Essays Submitted Here

(The ten articles previously published that are the core of the collection are, of course, not submitted here.)

Preface
Introduction
Postscript to "Frege on Truth"
Postscript to "Frege and the Hierarchy"
Postscript to "Sinning Against Frege"
Postscript to "Frege on Apriority"

Preface

I read Frege in graduate school, of course. But I became seriously engaged only when I began teaching his work in the 1970's at UCLA. Each year that I gave the course, I spent three quarters of the time on Foundations of Arithmetic. On my own, I devoted many hours to formalizing his definitions, doing derivations with them, and making handouts for students. Foundations of Arithmetic engendered enthusiasm, both in my attitude toward the teaching and in the students' response. Frege's powerful criticisms of alternative views made a deep intellectual impression, and his obvious struggles in developing his own positive positions on fundamental philosophical issues were inspiring. Frege's philosophy of mathematics was the center of the course. His papers in the philosophy of language came last, occupying only a quarter of the time and providing "glimpses beyond". I centered on the philosophy of mathematics because I thought it had to be understood, because UCLA's quarter system made courses relatively short, and because Frege's philosophy of language was taught in other courses. An unforeseen byproduct of this approach was that it helped me see how Frege's epistemology –his attempt to understand the nature of mathematical knowledge–lies at the philosophical heart of all his work, including his philosophy of language.

From the beginning, I found Frege's epistemology and his association of language with thought attractive. I was attracted to his rationalism. I found his concentration on thought and knowledge as expressed in language a welcome alternative to relatively narrow reflection on linguistic structures, which had been my starting point in philosophy. My developing philosophical commitments in the theory of reference and philosophy of mind made me sensitive to central points that Frege neglected or made mistakes on. But I thought that many of his positions were right and profound. I knew that there was much to be gained from reflecting on

his views on language, thought, and knowledge. The first two papers that I published on Frege (both in 1979) tried to work within Frege's point of view as much as possible. Yet neither paper employed as strict a historical methodology as I later came to believe was necessary for the most effective understanding and presentation of Frege's views.

At the time, I regarded both of these papers as "holiday" work–work to be done with the left hand so to speak, as a diversion from my main work in philosophy. I valued thinking along with Frege as I would value going through a mathematical proof or a physical workout. It seemed to be worthwhile in making one clearer-headed and stronger. And it was fun. In those first years, I did not see it as directly enhancing my own systematic work in philosophy.

I had had some training in history when I was in college. In coming up against ways in which Frege was philosophically foreign, I came to recognized that Frege should be studied as a figure in the history of philosophy. With the paper "Frege on Extensions of Concepts: 1884-1903" (1984), I tried to apply historical methodology in a more rigorous way.

While I was writing this paper, an event occurred that marked what I think of as a more substantial change in my approach to the study of Frege and to the history of philosophy. In one of the last revisions of the paper, before submitting it for publication, I came suddenly to see that Frege's rationalism guided his conception of sense in a much deeper way than I, or perhaps anyone, had appreciated before. I came to recognize how profoundly different Frege's conception of sense is from modern conceptions of communal linguistic meaning. The differences between modern views and Frege views about demonstratives and proper names–which I and others had previously recognized–came to seem only the tip of a very large, strange, and wonderful iceberg. Some of these differences are discussed in the last sections of that 1984 paper, in "Frege on Sense and Linguistic Meaning" (1986), implicitly in "Intellectual Norms and

Foundations of Mind" (1986),[1] and in subsequent papers on Frege. What I want to emphasize here, however, is not the content of my realization (the way that Frege's rationalism guided his conception of sense). I want to emphasize the effect that the realization had on me.

I retain an absolutely vivid image of the moment when the realization came to me. I was working alone late at night in a warmly, but dimly lit dining room–not my usual place for working on philosophy. The idea seemed to erupt like a sharp explosion and then to spread like a lava flow. I was completely absorbed. I was keyed-up, but in a way that did not affect a concentration that lent some weight, in retrospect, to the old metaphors of the intellect's being emancipated from time. It was the sort of moment of insight and discovery that one is granted only occasionally, but which sustains intellectual life–both in the thrill of the initial revelation and in the gradual realization and working out of consequences and connections. It is the intellectual counterpart of falling in love in a way that stays solid, develops, and deepens beyond the initial excitement. The experience was a heightened instance of how--for all the frustration of struggling through the difficulty of the subject, and all the tediousness of being careful to put things in order and avoid foreseeable mistakes–, philosophy can be both thrilling and life-sustaining.

The experience changed my attitude toward engagement with Frege, and indeed toward the role of the history of philosophy in my broader philosophical work. Studying Frege became not merely an exercise in intellectual hygiene and development. It became a means of philosophical discovery. There is much that remains to be understood about Frege. There are whole reaches of his thought that offer the possibility of breakthrough insights to students of his work. I invite the reader to join in the quest for further discovery.

In landing at UCLA after ejecting from graduate school, I was very fortunate to have three senior colleagues who had an exceptionally deep understanding of Frege. I had several helpful

conversations with Montgomery Furth when I was starting out. Furth's translation and introduction to The Basic Laws of Arithmetic and his "Two Types of Denotation" were invaluable in my early Frege education.[2] Deplorably, the former work is out of print. Furth's writing about Frege remains among the best introductions.

Alonzo Church was a sometimes ghostly, sometimes substantial presence in my early years at UCLA. Some of us used to joke that Church was Frege himself, having learned English--with his German origins thinly disguised by the overlay of a Virginia accent-- and having picked up more than a few inches and pounds. Church was not a person one conversed with. I audited his courses, read his work, and was influenced by his intellectual standards and by the power of his pragmatic rationalist point of view.[3] Often, experiencing the presence of a great intellectual has an effect that goes well beyond what mere reading or even listening could achieve. My experience of Church was of this sort. Church did as much as anyone in making Frege's work effective in twentieth century philosophy. His particular impact on my understanding of Frege was very substantial–in inspiration, in the formation of standards, and in developing instincts for Frege's ways of thinking. Furth and Church are no longer among the living. So my thanks to them must be correspondingly attenuated.

The third colleague who influenced my understanding of Frege remains a colleague. I have greatly benefitted from the intellectual example and historical-philosophical instincts of David Kaplan. Kaplan had been a student of both Church and Carnap. His early work was in formal Fregean semantics. He has maintained a pedagogical and systematic interest in Frege throughout his career. Although Kaplan disclaims using historical methodology, he reads as closely as anyone I know. His historical instincts about Frege are reliable and true. Repeatedly, I have found, by checking texts, that Kaplan's "off the cuff" claims about Frege go to the heart of

both large structure and subtle nuance in Frege's conceptions.

As any teacher must acknowledge, I have benefitted from the enthusiasm, questions, and insights of many students. Among these I specially think of Nathan Salmon and Marco Ruffino, both of whom went on to write on Frege.

I have also learned a great deal from Charles Parsons and Tony Anderson over the years. Both have been valuable interlocutors. The occasions on which we interacted are few in number but large in impact. More recently, I have been stimulated in genuinely fruitful ways by discussions of Frege with Christopher Peacocke.

Finally, I want to acknowledge more personal debts. My family supported and endured–chiefly my wife Dorli, but also my sons Johannes and Daniel and my parents Mary (now deceased) and Dan.

Footnotes

1. The Journal of Philosophy vol. 83 (1986), pp. 697-720.

2. Gottlob Frege, The Basic Laws of Arithmetic: Exposition of the System, translated and edited, with an introduction by Montgomery Furth, (Berkeley, University of California Press, 1967); Montgomery Furth, "Two Types of Denotation in Studies in Logical Theory, American Philosophical Quarterly Monograph Series, Monograph no. 2 (Oxford, Basil Blackwell, 1968).

3. I came to be an editor of Church's work, a task I began in the late 1970's: The Collected Works of Alonzo Church (Cambridge, Massachusetts; MIT Press, 200?). CHECK

Intro to Essays on Frege

List of things to do:
1) Every paper should be equipped with the pagination of the original article. This is a job to pay someone for.

Introduction

Gottlob Frege (1848-1925) is surely the great philosopher least known to the general intellectual public in proportion to his stature and his effect on philosophy. He was relatively unknown and unappreciated even by mathematicians and philosophers during his lifetime. Toward the end of his career, his contributions were promoted by Russell, Wittgenstein, and Carnap–and slightly later by Church. He came to be recognized as the fountainhead of mainstream philosophy in the twentieth century–the century's most important and influential philosopher. His main work was published between 1879 and 1903. It came to life and had its effect only subsequently.

There are many currents in twentieth century philosophy. What I called the "mainstream" is the approach to philosophy that has dominated the English speaking world for most of the last century. It has been central in Poland and Scandinavia, and is becoming increasingly prominent in the rest of continental Europe. In fact, it is showing every sign of becoming global. I think that this approach draws more of the best minds and carries on a more coherent and progressive discussion than any other within philosophy. It is this approach that owes more to Frege than to anyone else. For much of the twentieth century, this mainstream was called "analytic philosophy". I shall return to this label.

Although Frege is well appreciated in philosophy, his name and work remain unknown to the wider intellectual public. The reason for this obscurity is not hard to come by. Frege's writing is narrowly focused in philosophy of mathematics and philosophy of language. Some of it is technical. The subjects themselves are forbidding. Yet most of the passages of Frege's writing

that have had philosophical impact are not specially technical, and are not hard to read, as philosophy goes. Students new to his work warm to it quickly because of its crystalline clarity, the bold symmetries of its structure, and its appeal to intuition and common sense in discussion of abstract topics like the nature of number or the relation between thought and communication.

I mentioned that Frege's contributions are a source of what has often been called "analytic philosophy". The term is popularly associated with an emphasis on clarifying meaning and centering philosophical discussion on language and logic rather than on "reality". It is associated in the popular mind with expelling from philosophy terms or problems not fit for scientific or rigorous discussion, and with a negative attitude toward traditional philosophical methods and issues. It is commonly thought that this approach is too narrow to be of general interest. Hostility to philosophy, so conceived, is widespread not only in popular culture, but among the general intellectual public.

This popular view once had considerable merit. It still has some truth in it. Among the first appropriators of Frege's work were the logical positivists who dominated philosophy in the period between the 1920's and 1950. Although among Frege's three original promoters, Russell and Wittgenstein have greater individual philosophical stature than Carnap, Carnap was part of a movement–logical positivism–that had the broadest influence among those who first made use of Frege's work. Logical positivism has long been overthrown as a doctrine. Yet it cast a large shadow into the latter half of the twentieth century. Most of the charges of the preceding paragraph have some force against this movement and at least some of its successors. The charges are, however, one-sided and constitute a caricature, even of logical positivism.

The charges are one-sided inasmuch as they miss the genuine methodological progress that occurred in philosophy through the first half of the century. Owing much to Frege's

development of modern logic, his alliance of philosophy with scientific attitudes and norms, and his clarity in exposition and rigor in argument, the standards for philosophical discussion took a giant leap forward. The effect of this progress was to make philosophy a much more communal enterprise. Philosophical theses could be discussed more fruitfully. Reasons and criticism could be more easily assessed. Grounded agreement or disagreement became more common. Even though philosophy is not, in general, a science, it took on some of the best methodological characteristics of science.

The charges constitute a caricature inasmuch as they fail to recognize the seriousness of the basic problems that engaged the positivists. The positivists inherited from Frege an interest in meaning as expressed in language. Both Frege and his positivist successors were concerned to understand language and meaning because they saw such understanding as a new and promising route to understanding the nature of human knowledge. Frege and the positivists took human knowledge to be best exemplified by scientific knowledge. (As we shall see, however, they understood this point in very different ways.) This approach to philosophy through a consideration of the nature of human knowledge is, of course, traditional. It had dominated the subject from Descartes through Kant. Kant was a source of inspiration for both Frege and his positivist successors.

In broadest terms, Frege added two things to this tradition. He added a concentration on language in the expression of knowledge. And he added a recognition of the power of logic to illuminate the structure of language and its contribution to the expression of knowledge. Frege is responsible for establishing and developing modern logic–a huge achievement. He used logic to brilliant effect in investigating the structure of language and the ways it connects to the world and expresses thought. He used this investigation to better understand human knowledge. In this

respect, Frege should be seen as continuing and making major contributions to a central tradition in philosophy.

Frege's positivist successors used Frege's innovations. Yet they did so against a background of philosophical attitudes that Frege did not share. They differed with him, in absolutely fundamental ways, about both meaning and knowledge.

Although the positivists took up Frege's focus on language, they appended to it an ideology of exclusivism. The only meaning that might be of any cognitive value was, for them, scientific meaning. It is this ideology that led positivism to be aggressively deflationary about those aspects of philosophy, and culture that it could not assimilate to a scientific paradigm. Frege is specially interested in the language of the mathematical sciences. But his doctrine exhibits no such exclusivism. It exhibits no inclination to deflate philosophical problems that are not at the center of his scientific focus.

Whereas Frege's successors saw themselves as overturning traditional philosophy, Frege saw himself as continuing a philosophical tradition. Whereas his positivist successors applied reductionistic or deflationary attitudes to nearly all philosophical problems, Frege confined his reductionism to the attempt to reduce the mathematics of number to logic. He shows no special inclination to hold that reduction is a good method in philosophy generally. He shows no inclination to hold that the task of philosophy is to show that philosophical problems amount to less than meets the eye.

The positivists begin with the hypothesis that there is something cognitively defective about any meaning that is not the meaning of scientific language. They begin by demanding an explanation of meaning that can be distilled down to scientific elements. Notoriously, they tried to reduce meaning to procedure for verification. It is, however, the method of their approach that

I want to highlight. The notion of meaning was assumed from the beginning to need reduction, deflation, reshaping, or explanation in terms that were "scientifically acceptable". This attitude has continued in philosophy well after the demise of positivism. In fact, forms of scientism, reductionism, and deflationism have remained prominent in philosophy since the positivist initiative.[1] Frege is certainly concerned with ways in which ordinary language can be an obstacle to the progress of science. But his more open or pragmatic approach to philosophy stands clear of these trends set in motion by the positivists. I believe that his approach will remain valuable as its scientistic-reductionist-deflationist off-shoots diminish in prominence.

Frege postulated two types of "meaning": a) reference or denotation (Bedeutung), a relation between language and its subject matters, and b) sense (Sinn), the way of thinking relevant to the truth or falsity of the thought that is directly associated by the individual language-user with his linguistic expressions. Frege provided deep and lasting accounts of the structures of these types of meaning. He showed no interest in explaining them in terms that are more scientific according to some pre-conceived standard. Nor did he think that they have to be ignored or dispensed with in a final science. I believe that he would have taken senses to be part of the subject matter of a genuinely public, scientific psychology and linguistics as these disciplines are now being pursued. I see no textual basis for holding that he thought that the semantics of a scientific language is not every bit as factual as any other special science. His work on the two types of meaning was the historical basis for all major subsequent work in semantics-- in philosophy, mathematical logic, and linguistics.

Frege's positivist successors differed with him fundamentally about knowledge as well as about meaning. Frege's fundamental motivation, like that of the positivists, was to understand human knowledge.[2] Whereas positivism, in fact most post-Fregean mainstream philosophy until

the last decade or so, was aggressively empiricist, Frege was a rationalist. That is, positivism was centrally motivated by the view that all genuine knowledge depends for the force of its warrant or justification on sense experience. Frege believed that knowledge in logic and arithmetic depends for justification only on reason.

Logical positivism rests on two doctrines: a) that statements in mathematics and logic are analytic in the sense that their truth does not depend in any way on relations to a subject matter but depends purely on their meaning, and b) that the cognitive meaning of a statement consists in its method of verification. (Roughly, cognitive meaning is meaning that can be rationally construed as involved in a true or false statement.) The second doctrine ties cognitive meaning to science. The first maintains that logic and mathematics do not constitute genuine knowledge or genuine science: Knowledge of a subject matter is restricted to the natural sciences. Both doctrines were overthrown, I think decisively, in mid-twentieth century.[3]

The positivists took their paradigm science to be physics. Frege's paradigm science was mathematics. Frege thought of proof in mathematics, ultimately in logic, as the paradigm type of justification or "verification", although of course he recognized inductive justification in empirical science. The positivists took proof in mathematics to be merely an instrument within natural science with no independent status. Proof in pure mathematics did not count for them as justification of any genuine knowledge at all. They took experiment in natural science to be the paradigmatic justification.

These differences regarding justification and knowledge are associated with differences about ontology– about what exists or what "has being". The positivists took mathematics and logic to be true independently of a subject matter. This is the first of the two doctrines mentioned above. They helped produce a philosophical climate in which suspicion of abstract

entities–whether numbers, functions, or senses–was axiomatic for many philosophers. Frege exemplifies the natural ontological attitude of working mathematicians, a relaxed ontological Platonism. He took numbers and functions to be a subject matter of mathematics and saw no scientific reason to hold that they occur in space or time. He extended this attitude to his account of sense or cognitive value. I believe that Frege's Platonism is a source of both philosophical insight and philosophical excess. I shall return to this matter.

A further difference between Frege and his most influential successors lies in their views about logic. Positivism was committed to seeing logic, like mathematics, as lacking a subject matter. Often this commitment took the form of seeing logic as a formal tool for aiding inference in the empirical sciences, but as having no "substantive" content of its own. Frege saw logic as a scientific language expressing knowledge of a subject matter, in this respect like any other science. He regarded its meaning or content as just as substantive as that of any other scientific language. He took its truths to be as just as dependent on relation to a subject matter as the truths of any other science. In fact, he saw logic as the most general science of "being". Here too Frege is part of the main tradition in the philosophy of logic. I believe that he used this point of view to make fundamental contributions to understanding logical truth and logical consequence.

I mentioned that Frege is the source of what is commonly called "analytic philosophy". I indicated a preference for the term "mainstream twentieth-century philosophy". I think that "analytic philosophy" is a term quite appropriate to the work of Frege's positivist successors but at best misleading as applied to Frege's own work. It has become a commonplace within philosophy to note that the term "analytic philosophy" is now used so loosely, and has come to apply to such a diverse discipline, that there is no general substantive characterization of it. Only historical and extremely broad (nearly uninformative) methodological characterizations seem to

fit. "Analytic philosophy" can be appropriately construed as a proper name not a description. Yet in its early applications the phrase had solid descriptive aspects. It suggested a central concern with meaning cut off from metaphysics or "reality". It suggested an approach to philosophy through a method of analysis. It suggested the prominence of analytic truths among philosophical results. These suggestions remain in the general intellectual consciousness.

I have already criticized the popular idea that philosophy during this period is exclusively concerned with linguistic meaning. Frege, Russell, and the positivists were concerned with linguistic meaning because they saw it as the key to understanding human knowledge.

It is true that parts of philosophy of language and philosophy of logic became relatively autonomous in the latter part of the twentieth century. Language and meaning came to be studied for their own sake. Parts of philosophy are now continuous with semantics in linguistics. Parts are now continuous with semantics in mathematical logic. These are healthy philosophical developments with ample precedent in the history of philosophy. Philosophy has always been midwife to new sciences. Frege deserves the largest credit for both of these developments, even though they occurred after he had done his main work.

But even now, the main line of philosophical development continues to link understanding language with a larger range of philosophical topics. These topics include the way that language expresses knowledge, the relation between meaning and communication, the transmission of knowledge through communication, the way that understanding meaning aids in understanding mind, the way that understanding particular discourses contributes to understanding the topics of those discourses–from aesthetics and ethics to scientific explanation. In these traditional philosophical enterprises, Frege's work on language has been a stepping stone to understanding the whole range of philosophical problems, including traditional philosophical problems.

The popular picture that concern with linguistic meaning in analytic philosophy proceeds independently of concern with "reality" is a version of the mistake embodied in the positivist doctrine that logic and mathematics are analytic. This is a mistake that Frege did not make. His conception of sense and his semantical doctrines are inextricable from his concern with the nature of "reality" or being. Here again Frege's revolutionary innovations are in the service of a type of philosophical inquiry as old as Plato and Aristotle.

What of Frege and "analytic method"? There is a very broad sense in which Frege's method is analytic. He seeks to isolate basic concepts and basic principles in trying to demonstrate logicism–the view that the mathematics of number is reducible to logic. This enterprise of seeking basic concepts and principles is again common to many of his philosophical forebears–Plato, Aristotle, Descartes, Leibniz, Kant.

It would be a fundamental mistake, however, to see Frege as carrying out this method by analyzing definitions of words through sheer reflection on the words or through focusing entirely on common linguistic usage–even usage within science. In fact, Frege's way of trying to understand linguistic meaning, linguistic structure, and fundamental principles is notably synthetic. It is fundamental to his attempt to understand meaning that his investigation proceed by considering the whole range of inferences that are expressed in the use of any particular piece of language. He sought understanding through holistic, systematic reflection on the discursive uses of language.

Frege does not assume that existing usage is determinative of what language expresses. He allows any element of simplification, and any piece of rational insight or discovery about the subject matter, to be potentially relevant to understanding the nature and structure of senses and denotations–his two types of meaning. Reflection on whole systems of inferential and other

cognitive activity is fundamentally a synthetic enterprise. Relevant systems are not treated as given (there to be simply analyzed) independently of ongoing inquiry. So conceiving Frege's method as analytic is at best deeply misleading.

The third descriptive connotation associated with the term "analytic philosophy" is that philosophy so named is concerned with analytic truths. Frege does have a concept of analytic truth. But he explains it simply as truth provable from general laws of logic. Analytic truths are, definitionally, just a subclass of logical truths. The notion is associated with a certain form of justification–proof from logical laws. Relative to the issues associated with analytic truth in the twentieth century, his concept of analytic truth is philosophically non-committal.[4]

The sense of analytic truth that the positivists urged and which entered into the popular conception of analytic philosophy is different from Frege's. That notion of analytic truth is expressed in the first of the two positivist doctrines. It is the idea of a truth made true by meaning alone, independently of any subject matter. It is the notion employed by the positivists to cut off mathematics and logic from epistemic and metaphysical inquiry. The positivist regarded empiricism as a fundamental philosophical commitment. Empiricism claims that all genuine knowledge of any subject matter is warranted through sense experience. If logic and mathematics were true independently of any subject matter, they could not threaten empiricism. And they could not engender metaphysical questions about the nature of mathematical entities. Many positivists held that where philosophy made true statements, as opposed to practical recommendations, those statements are also analytically true–in the sense that their truth does not depend on a subject matter.

Frege's philosophy recognizes no analytic truths in this sense. His primary interest lies in the nature of human knowledge of mathematics. He engages in deep reflection about the nature

of the objects and functions that this knowledge is about. He takes the knowledge that he gains through reflection on meaning (knowledge of sense as well as denotation) to illuminate the nature of human thought, and indeed the nature of all knowledge. So his vision of philosophy could not be further from the picture advanced by the positivists. They pictured philosophy as simply producing practical recommendations and analytic truths, which delimit its own ambitions and scope. Frege's philosophy is a philosophy of inquiry and discovery.

I have emphasized some fundamental ways in which Frege differs from the most influential appropriators of his work, and from the popular conception of the approach to philosophy that he helped initiate. One might think that these are aspects of Frege's work that simply got left behind. I believe that this is not true. The matter is, however, complex.

The dissolution of the distinctive spirit of positivism has been a much slower process than the collapse of its letter. That spirit is still alive. But it is not dominant. It contends with eclectic, pragmatic approaches to philosophy driven more by the diversity and difficulty of philosophical problems than by deflationary ideologies.

Conceptions of meaning, particularly those associated with the problems that Frege introduced his notion of sense to solve, are still in flux in the wake of the failure of the verificationist theory of meaning. There are approaches not moved by the positivist spirit. One such approach is that of taking semantical notions roughly at face value and using them as refined theoretical primitive notions in a semantical theory. This was Frege's method.

The implications of the failure of the positivist conception of analytic truth have not been fully digested. But the appeal to analyticity, in the positivist's sense, is no longer prominent in epistemic and ontological enterprises. Moreover, Frege's rationalist conception of knowledge of mathematics and logic has undergone a notable revival. An openness to inquiry, which his

philosophy exhibits, has begun to replace the predominantly reductionist and deflationist spirit that marked much of the philosophy influenced by his work. I believe that the aspects of Frege's philosophy that I have emphasized will prove to be longer-lived than contrasting counterparts. I believe that these aspects will be a continuing source of inspiration in philosophy.

Frege's philosophy should be of intellectual interest to the broader intellectual public in any case. He set an example for philosophical reasoning and method that had the largest effect in producing a very deep change in the standards for philosophical discussion. This change has seeped into every area of philosophy, not just the areas that Frege and his immediate successors concentrated upon. Its effect has been to make a larger portion of serious philosophy accessible to a broad intellectual public than perhaps at any other period since the period of the Greeks.

Of course, the most original, serious philosophy is never easy. The subject matter is abstract. Progress often requires precision and a precisely understood vocabulary. One simply has to read serious philosophy more slowly than many other subjects. But much of Frege's writing lies in the tradition of Plato, Descartes, Hume, Russell–presenting difficult matters in a style that can be grasped by intelligent non-specialists. Frege's example changed standards for clarity of expression and argumentative rigor.

Historical interest in his work is justified by a reason that should motivate all historical interest: It is a means to better self-understanding and better realization of ones own pursuits–in this case philosophical and, more generally, intellectual pursuits.

Then of course, Frege's philosophy is intrinsically interesting. It is old enough to be fruitfully foreign. Yet it has a currency and vitality that belies its age. The essays collected here explore aspects of Frege's philosophy that are of current and, I believe, long-term philosophical interest.

Before providing a more specific introduction to the philosophical orientation of this collection of essays, I would like to discuss my motive and method in writing them.

My motive has always been philosophical. Frege has an obvious and special relevance to philosophical work in our time. One way to gain a perspective that distinguishes the deep from the superficial is to reflect on great thinkers of the past. Frege had deep and lasting insights that are fundamental for the theory of knowledge, philosophy of language, philosophy of mind, philosophy of logic, and philosophy of mathematics. Understanding a great philosopher in depth helps deepen one's own thinking. Some of the most important philosophical values of studying Frege emerged for me only after years of study and reflection. I hope to communicate some of these values.

My method in writing about Frege has been, almost from the beginning (see the Preface), steadfastly historical. History of philosophy is a branch of philosophy as well as a branch of history. Approaching history as part of an ongoing philosophical enterprise, however, carries dangers that are more acute than in other types of history. In studying the great figures of the past in the service of truth in philosophy, it is easy to assimilate their ideas to more recent ones or to make mistakes in criticizing them because one overlooks differences in their terminology, aims, or background assumptions. These tendencies are especially easy to fall into if one is too quick to argue with them or treat them as contemporary interlocutors or allies. To appreciate the full depth of a great philosopher of the past, one must be patient. One must maintain a reflective listening attitude that goes beyond what is needed to understand a contemporary.

There is, first, temporal distance to compensate for. Presuppositions change. What seems obvious changes. Reader-expectations change. Familiar terms change meaning or at least

connotation. One can master these changes only through systematic reading and rereading, and through the consistent exercise of historical as well as philosophical judgment and perspective.

There is, further, the depth of genius to accommodate to. Appreciating the size, richness, detail, and originality of the conceptions of a great philosopher again requires patience in listening, in reflecting, and in willingness to read and reread in systematic and comparative ways.

Given the dangers of parochialism, of mis-measuring historical distance, and of underestimating philosophical genius, I have found it important to support my exposition through substantial citations of texts. This can seem tedious and overly scholarly. It is, however, an important control. Attempting to "tell a story" that is not tied down at every turn to textual evidence commonly becomes an expression of the story-teller's own ideology and commonly fails to appreciate the complexity, depth, and foreignness of a great philosopher's work.

Of course, given my philosophical motivation, historical exposition is guided by my philosophical interests. Historical method can help counteract tendencies toward glib disagreement and ideologically based "stories" based on incomplete understanding. But it has its own dangers. The expenditure of energy that goes into getting a figure historically right can sap the historian's philosophical energy. Historical work can fall into slavish agreement or uncritical reporting. I hope to have avoided these pitfalls. Although I present Frege's thinking without constant evaluation, I have definite views about what is right and what is wrong in Frege. (I should add that these have changed in some ways with deeper understanding of his work.) Most of my differences, and the deeper, more subtle ways in which Frege has been a positive influence on me are, I think, best left to venues in which I present my own philosophical work.

Historical perspective is, however, served by evaluative comment. Since I see my reflections on Frege as an adjunct to my own philosophical enterprises, I cannot resist connecting

the two at least at some of the most central junctures. I hope that these essays will be read in the light of, and as part of, my own philosophical contributions. Still, in these essays I have tried to train the historical focus on Frege. Whether I have supplemented this focus with philosophical judgment in an illuminating but unobtrusive way is for each reader to decide.

The essays collected here do not in any sense comprise a complete view of Frege's philosophy. There are many aspects of his work that I do not touch. Some are mentioned as background or drawn on but taken for granted. Many have been well discussed by others. The essays center on issues in Frege's work that are of special philosophical interest to me.

In some cases the essays will be more difficult to read than Frege's own work. Understanding them well benefits from having at least some background in Frege. In discussing pros and cons of interpretation and offering substantive evaluation, they are sometimes more complex than at least the surface of Frege's own writing. Still, I hope that patient, slow reading, will find them forthright and clear. I hope that in combination with more direct reflection on Frege, they will yield philosophical insight and stimulation in the reader.

The essays center on Frege's views about "meaning" and knowledge–the two issues cited earlier on which Frege differs most deeply from the logical positivists. This neat categorization is made more complex by the fact that Frege conceives of "meaning" as being of two types (Bedeutung and Sinn). It is also made more complex by the fact that his investigation of meaning is driven by a certain conception of truth as an aim that underlies language and science, and by the fact that his investigation of truth and meaning are intertwined with his views about knowledge.

After a brief overview paper, "Frege" (1991), the essays are divided, somewhat awkwardly, into three sections. Section I concerns Frege's investigation of structure–the

structure of language and thought, which he sees as a part of the structure of knowledge. Frege's work on truth as the aim of logic and science and his account of the structure of "meaning" are the seeds from which all his philosophical contributions grow. I am interested in Frege's investigation of structure because it exhibits his method of coming to understand linguistic and epistemic structure. An offshoot of this investigation is his account of the structure of sense. Indeed the very conception of sense as cognitive value is understood ultimately in terms of its role in aiding pursuit of truth. Section II deals primarily with Frege's conception of sense as cognitive value. Section III centers on Frege's rationalist conception of knowledge. Obviously, the three sections should not be regarded as compartmentalized. In Frege's own work, the issues are interwoven. So reflection on the topics of any one section should deepen one's understanding of the topics of the others.

I would like now to make some more specific philosophical remarks about Frege's contributions, as discussed in these three sections. These remarks are meant to serve primarily as philosophical orientation on the largely historical accounts that the essays offer.

Section I: Truth, Structure, and Method.

The project that drove Frege's life work was an attempt to establish logicism. Logicism is the view that the mathematics of number can be reduced to logic. Establishing logicism requires defining the primitive mathematical expressions or notions in terms of expressions or notions of pure logic, and then using the principles of logic together with the definitions to prove the axioms and theorems of the relevant mathematics. Frege hoped to show that the mathematics of number–principally arithmetic and analysis–could be thus reduced to logic. He regarded geometry as a different type of mathematical subject, not reducible to logic.

Frege conceived his logicist project as a contribution to the theory of knowledge. He

wanted to explain the fundamental character of our knowledge of arithmetic and analysis. Although this philosophical motivation is very clear, Frege proposed to carry out his project with the rigor of a project in mathematics. In doing so, he established standards for rigor unprecedented in philosophy and hardly precedented in mathematics.

Frege realized that to carry out the logicist project he needed a precise logical system. To this end, he developed modern first-order and higher-order logic. This was by far the largest step forward in logic since Aristotle. In fact, to oversimplify for effect without fundamentally distorting the historical or philosophical facts, this was the first major step forward in logic since Aristotle. With it Frege completed first-order and second-order logic proper–logic considered independently of its meta-theory.[5] Frege's development of logic is one of the great achievements in intellectual history.

Frege also realized that to carry out the logicist project, he needed to understand the logical structure of principles of mathematics and logic in a sufficiently definite way to be able to carry out the relevant proofs. This realization led Frege to three of his deepest insights. These insights can be usefully seen as part of the following idealized reasoning (though I do not claim that Frege used precisely this reasoning). Thoughts or principles are most perspicuously expressed in sentences. Understanding the logical form or logical structure of sentences requires understanding the logical structure of their component parts. Logical structure is revealed in the way that good deductive inference hinges on structure. So understanding logical structure of sentences and their component parts depends on understanding the structure of good deductive inference. Understanding the structure of deductive inference depends on systematic reflection. For logical inference can combine <u>any</u> sentences or thoughts in a single argument, and by looking at numerous combinations one recognizes structural patterns that otherwise would not be salient.

So to understand logical structure of sentences and their component parts, one must reflect on the way that sentences enter into a wide range of inferential combinations. The point of inference is to preserve truth in making transitions from true premises to true conclusions. So in reflecting on linguistic structure as it is revealed in inference, one should focus on the contributions of elements in such structure to determining the truth of sentences and to preserving the truth of sentences in inference. So to understand the structure of sentential parts, one should reflect systematically on their contribution to conditions under which sentences count as true, and thus on their contribution to determining conditions under which truth is preserved in deductive inferences.

The three key insights are present in this sequence. First, thought is fruitfully understood by reflecting on language.[6] Second, one understands the structural nature of thoughts and components of thoughts not by taking those natures for granted , nor by relying on simple intuition about grammar or thinking, nor by invoking general philosophical dicta, but by reflecting discursively on a large number of inferences among sentences. Third, the key to understanding such structure lies in understanding the contribution of such components to determining truth conditions and to preserving truth in deductive inference.

These three insights came to dominate twentieth-century reflection on language and thought. They have proved to be fruitful and genuine. They have had their effect in tandem. The first insight led to the fruitful work in philosophy that I mentioned earlier. It also led to explosive developments in linguistics, psycholinguistics, and cognitive psychology. But it would not have had its effect if it had not been linked with the second and third.

The second and third insights had already been enunciated by Kant, at least in germ.[7] Kant held that concepts are essentially predicates of judgment. He held that judgment aims at truth and is essentially propositional. Thus he regarded components of thoughts as having their function

and structure only in the context of propositional judgment, and inferences among judgments.

It is likely that Frege's insights gained something from his exposure to Kant. But Frege's insights in this domain went deeper than anything one finds in Kant, or anyone else before Frege. Frege combined the second and third insights with the first. He centered his reflection on thought in an investigation of language. This provided reflection with a concreteness of application not easily obtained by other means. Frege had a full, precise logic to work with. Through this means, he gained a grip on the nature of numerous logical inferences that no one before him had. Frege applied the method of investigating the sub-structure of thoughts and sentences in a systematic and detailed way. This application made the insights come to life in accounts of the contributions of specific structures to specific inferences.

In The Foundations of Arithmetic (1884) Frege enunciates an idea that is closely associated with the second of the three insights. This enunciation is what is known as his context principle. In fact, writing as if he were stating a single principle, Frege presents, at different places in the book, three non-equivalent formulations. These are, I believe, correctly seen as three different principles.[8] When one takes account of Frege's later (1891) distinction between sense and denotation, each of these three formulations divides into two further non-equivalent formulations. In the end, I think it correct to say that Frege believed in at least six "context principles". Although Frege's formulations are terse and although his application of the principles leaves room for interpretation, I shall state what I take the principles to be.

The first two principles are methodological:

> Always seek to understand the denotation of a word not in isolation but in the context of its role in a sentence.
>
> Always seek to understand the sense of a word not in isolation but in the context of its role in a sentence.

The second pair of principles concern necessary conditions:

A word relates to a denotation only through its having a role in a sentence or in sentences.

A word relates to a sense only through its having a role in a sentence or in sentences.

The third pair concern sufficient conditions:

If a word is a component (under structural analysis) in a true sentence, it has a denotation; and the denotation is fixed through its contribution to the sentence's being true and to the preservation of truth in good deductive inference.

If a word is a component (under structural analysis) in a true sentence, the word has a sense; and the sense is fixed through its contribution to the cognitive value and its role in contributing to the cognitive aspects of good deductive inference.

The first pair simply states the method of investigation that I characterized earlier. The second pair provides a philosophical rationale for the method. The third pair–particularly the principle that concerns denotation–provides a philosophical basis for understanding theoretical reference to abstract entities and other theoretical entities of science. I would make certain qualifications on the principle before accepting it. But I think that it is deeply insightful inasmuch as it locates ontology in the evaluation of theories. It implicitly rejects philosophical requirements on reference or denotation that go beyond standard methods for evaluating the truth of sentences or theories. For example, the principle implicitly rejects the requirement that to make reference to an object through thinking about it, one must have a mental image of the object, or bear a causal relation to the object. It rejects such requirements unless they are motivated as requirements on the evaluation of the truth of sentences or on the evaluation of theories containing the sentences. I believe that all six principles are, with relatively minor qualifications, sound.

The third of the three insights is associated with Frege's recognition that judgment (or belief) and assertion– which aim at truth–are key to understanding thought and language. Frege's

logical work is motivated in a remarkably single-minded way by his attempt to understand principles governing truth. He sees logic as codifying such principles because logic attempts to understand preservation of truth in argument. The centrality of these motivations is the topic of both the short paper "The Concept of Truth in Frege's Program" (1984) and Part I of "Frege on Truth" (1986). The 1984 paper is essentially extracted from the 1986 paper. I include it because I think it brings to focus two major instances in which the depth of Frege's concentration on truth in motivating his views is underestimated.

One instance is the attribution of the Church-Gödel argument that all true sentences denote the same thing to Frege. This elegant and fascinating argument was certainly inspired by Frege's reasoning. But the attribution of it to him misses how central his conception of the centrality of truth in logic is. The other instance is Dummett's well-known criticism of Frege's treatment of sentences as denoting truth-values. This criticism underestimates how carefully Frege distinguishes sentences–vehicles for assertion–from names in his logic. I discuss this matter in more detail below.

Frege's three key insights led to further insights. They led first to a working conception of truth-conditional semantics. As mentioned earlier, Frege's practice, and many of the details of his work, have remained mainstays in semantics worked out in philosophy, linguistics, and mathematical logic.

Frege was probably the first to emphasize, in the context of a systematic study of language, the creativity of language use. He noticed that everyone uses new sentences every day. In effect, we are in principle competent to use and understand an infinity of sentences. He realized that this was possible only because the sentences were built out of a finite number of building blocks and a finite number of formal principles of construction. This insight reappeared

in Chomsky's work and was applied in understanding the grammar of natural language. Frege was centrally interested in the logical form and semantics of a language suited to science. These basic insights, and the example of his practice of finding structure through comparing numerous sentences and their relations in inference, underlies the main development of the scientific study of all language. This study has deep implications for understanding the nature of thinking.

Frege's analysis of structure in terms of its contribution to specifying conditions under which a sentence is true led to his distinction between propositional content (the bearer of truth or falsity) and force (the attitude or speech act that makes use of the content). For example, Frege drew a clear distinction between the logical operator negation and the speech act denial, and between a thought content and the judgment (or assertion or supposition) of the thought content. These distinctions have been a basis for more detailed theoretical distinctions between semantics and pragmatics.

The focus on truth conditions helped Frege draw his famous distinction between sense and denotation. By reflecting clearly on what conditions in the world make sentences or statements true, he realized that the cognitive value associated with component expressions must differ from their denotations or references. Frege associates his notion of sense essentially with determination of truth conditions. No alternative conception of sense or meaning has been any stronger or more fruitful.

Frege's notion of sense has been controversial. As I will explain below, much of the controversy rests on misunderstanding. But aspects of his use of the notion that are certainly mistaken. Frege makes commitments in his theory of thought, in which his notion of sense is a central feature, that cannot ultimately be sustained. Nevertheless, the explanatory role that Frege gave his notion of sense--that of representing cognitive value--must, I think, be filled by some

substantially similar theoretical notion.

Frege associates his notion of sense, partly but essentially, with determination of truth conditions. No alternative conception of sense or meaning has been stronger or more fruitful. Frege used the focus on truth conditions to help distinguish between what is part of the cognitively relevant "meaning" of a sentence (both denotation and sense) and what is part of the associated tone, or other more loosely related cognitive or linguistic associations with the sentence. This distinction was taken up by most other subsequent theorists of language.

Frege's second and third insights not only opened fundamental distinctions in semantics and in the epistemology of language use. They helped him make two fundamental contributions to the meta-understanding of logic.

One is a contribution to the understanding of the fundamental notions of meta-logic– logical validity and logical consequence. Frege provided a non-modal semantical explication of the way logical truth and deductive logical consequence depend on the form of sentences and their component parts. Frege's insights were taken up by Tarski and Skolem to provide the first mathematically rigorous explication of the traditional non-modal notion of formal consequence (and logical truth explicable in terms of formal logical structure). Frege's explications are not as systematic as Tarski's. But they are clearly in a meta-logical tradition explicates logical truth and good deductive inference in terms of form and structure rather than in modal or epistemic terms. Frege is a central figure in a tradition that includes Abaelard, Scotus, and Bolzano, and which construes logical truth and logical consequence in these intuitive terms.

Most representatives of this tradition hold that logical truths and good deductive inferences are necessary and can be known in ways that are justified independently of sense experience. The explication of logical truth and good deductive inference in this tradition relies,

however, on logical form and logical structure rather than on necessity or some analog of apriority. The notion and role of explication vary from one representative of this tradition to another. Frege regarded semantical explication as non-fundamental from an epistemic point of view. But he took semantics seriously. In fact, he was the first author to develop semantics in a systematic way–the way that eventually flowered in its full mathematicization. I see no reason to think that he doubted that semantics yields knowledge.

In any case, it seems clear that Frege not only belongs to the tradition of understanding logical truth and logical consequence in formal or structural terms. He made fundamental contributions to that tradition–given his clarity about logical form and the semantical structure, his association of logical form and structure with maximal generality (rather than modality), and his association of semantical structure with truth.[9]

The other fundamental contribution to meta-logical understanding is a contribution to untangling the traditional problem of predication. This problem is that of explaining the difference between predication and the relation between a predicate and what it predicates. Plato pointed out that if in a sentence like "Theaetetus sits", the name "Theaetetus" stands for the man, and "sits" stands for the property (or form) of sitting, there is a problem of explaining how the sentence differs from a list in which expressions standing for the man and the property of sitting occur in succession. Plato observed that introducing a third entity–participation or inherence–does not by itself solve the problem if it becomes simply one more entity to be stood for. (So the list Theatetus, inherence, sitting gets us no further.) Plato makes the obvious point that the verb "sits" makes a different grammatical contribution to a sentence from any contribution made by its nominalization, "sitting". As far as I know, Plato did not take the issue much further. Understanding the peculiar nature of verbs (or more generally, predicates) in a

systematic way remained a problem that continued to worry philosophers through the centuries.[10]

It would seem to follow from the last point I attributed to Plato that even if "sits" and "sitting" relate semantically to the same entity, and even if the relation is the same, the grammatical standpoint from which they relate to the entity must be different. What they do with respect to the entity within the sentence is significantly different. That is to say, whether or not "sits" and "sitting" (or "the property of sitting") stand for entities, and whether or not they stand for the same entity, their roles in the sentence in connecting the relevant entity (if any) to other entities are different. Whether these different roles are bear different semantical relations to the subject matter, or whether (on the contrary) they bear the same relations from a different grammatical angle, seems to me a secondary issue. Some have thought that one must hold that one or both of such words do not stand for any entity at all. Some have held that they stand for different entities. I believe that none of these positions in themselves illuminates the original problem. The problem lies not at the level of the entities stood for but in the grammatical or logical roles of the expressions (or thought components) within sentences or thoughts.

The problem is to explain the role of predication in such a way as to distinguish it from whatever semantical relation a predicate bears to the world. The predicate bears whatever relation it bears to the world regardless of whether it is predicated of something. "Sits" bears its semantical relations--stands for the same property or is true of the same objects--regardless of what sentence it occurs in. When "sits" comes into combination with "Theaetetus" in a sentence, it does something more: it attributes a property of (or to) an individual, or otherwise constitutes predication. How is this "something more" to be understood in a systematic way?

The question is so close to bedrock that it is hard to know what sort of explanation would be illuminating. But Frege found a way to make a fundamental contribution to answering this

question. He accepted the point that the semantical relation associated with predicates differs from the semantical relation associated with names, though he saw both as types of denotation–as types of expression-subject-matter-relation.

Frege then mobilized two ideas. First, he explained the semantical relation associated with predicates in terms of the relation between a functional expression and a function. This idea enabled him to explain predication in terms of functional application. A function word stands for the same function at all times, but when it is grammatically combined with a name (or other expression for a input or argument of the function), the combination stands for functional application and yields the value of the function. Thus Frege explained predication in terms of functional application. The distinction between merely standing for something and predicating that something of something else was illuminated through a mathematical operation that has firm and independent explanatory power.

Second, Frege associated predication with its use in judgment and assertion, consequently with their objective or aim–truth. The function of judgment and assertion is to arrive at or present the truth. Frege associates those functional applications which for him constitute predications with the functional value, truth. So the application of the function that "sits" functionally-stands-for to the man that "Theaetetus" stands for yields truth as the value of the functional application. Of course, some predications fail their objective. Such predications yield falsity. Frege illuminated predication by associating it with a mathematical operation integrated into a semantical account that illuminates the point of predication, and linguistic use more generally.

Although Frege is correctly credited with deep insight here--an elaboration of the second and third insights that I discussed earlier–, he is commonly seen as having made two serious mistakes. Frege seems to have thought that in view of the deep difference between the semantical

relations associated with predicates and names, the two cannot bear their relations to the same sort of entities. Thus he would hold that "sit" and "sitting" (or "the property of sitting") cannot stand for the same entity. It is sometimes also suggested that he thought that if names and predicates are to bear semantical relations to the same entities, one would have to treat sentences as lists of names or lists of predications.

These would be fallacious inferences. It certainly does not follow from the deep differences between names and predicates as regards their grammatical roles that a name cannot stand for the same entity (a function) that a function-expression functionally-stands-for. A difficulty in Frege's position did emerge when he committed himself to asserting the German analog of the sentence "The concept (or function) horse is not a concept (or function)." Since the subject term "The concept horse" has the grammatical role of a name, Frege thought that it could not stand for a concept (or function). This is deeply counterintuitive. I believe that this constitutes one of Frege's most serious mistakes. Church showed, in his calculus of lambda-conversion, that there is no reason why a syntactically unsaturated expression–one with no open argument places or free variables–cannot denote a function, including a concept. The key point is to distinguish their grammatical roles.[11]

The second common criticism of Frege is that in taking predicates to stand for functions whose values are truth or falsity, he assimilated sentences to names–names of the values or outputs of predicational functions. It is commonly pointed out that it is artificial to take truth and falsity to be objects and even more strange to take sentences to be, in effect, names of those objects. Over many years, Michael Dummett held that Frege's views here do constitute an assimilation of sentences to names or singular terms. He insisted that Frege's assimilation constitutes a fundamental error. He maintained that the assimilation is incompatible with Frege's

own insight, elaborated in the context principles, that sentences are deeply different from components of sentences in their use and place in the language. In expressing thoughts that are true, or at least thoughts that are used to aim at truth, sentences are central to judgment, assertion, and inference in ways that singular terms are not.[12] Perhaps because they accord with a sense of the unnaturalness of taking sentences to denote truth values as objects, Dummett's criticisms seem to be widely accepted and often repeated.

I believe that I have shown in "The Concept of Truth in Frege's Program" (1984) and in Parts II and III of "Frege on Truth" (1986) that Dummett's criticisms are quite unsound. It is true that names and sentences are semantically similar in that they both denote objects. However, Frege maintains, even within his logic, a significant distinction between sentences and names. What Dummett thinks of as sentences are in Frege's actual logic *treated* as nominalizations of sentences. In fact, when Frege glosses unasserted names of truth values like (like "2 + 3 =5") in The Basic Laws of Arithmetic, he uses nominalizations like "2 + 3's being equal to 5" or "the true value thereof that if something is the square root of one then its not being the fourth root of one". (See, for example, sections 5, 8, 12, 13 of The Basic Laws of Arithmetic.) Thus what Dummett thinks of as "snow is white" is understood in Frege's theory as "snow's being white". "Snow's being white" does name a truth-value, and "white" does designate what Frege calls a "first-level concept"–a function from individuals to truth-values. But in Frege's theory the real occurrence of sentence-forming predication, an operation crucial to sentences' being assertable, lies in the occurrences of the horizontal sign, which translates as "is the true". It is a deeply significant fact, which Dummett completely overlooks, that the only expressions that are judgeable in Frege's theory (i.e. the only ones to which the vertical sign, the judgment stroke, can be applied) are expressions that begin with the horizontal sign.

So only expressions like "Snow's being white is the true" are assertable–not expressions that we would normally take to be formalized as sentences. Thus Frege takes "W(s)" to be a nominalization. All genuine sentences in his logic contain "is the true" (the horizontal sign) attached to nominalizations or other singular terms (like "7"). The horizontal makes any saturated expression into a sentence. Only expressions beginning with the horizontal sign (true sentences) are assertable. The horizontal sign is a component necessary for genuine predication in Frege's logic. In a sense, it is the only genuine predicate in his system. Thus in this subtle way, Frege's logic incorporates the conceptual point that strictly speaking only sentences are assertable. Dummett's famous criticism rests on a straightforward misunderstanding of Frege's theory.

I think that taking "is the true" as a component in all predication–indeed, in a sense, the only sentence-making predicate–is artificial. In fact, I think that there are substantive objections to this view. Frege's deep insight into the structural parallel between singular terms and sentences stands whether or not one goes along with his view. The structural parallel is that a sentence's truth value, like a singular term's denotation is a function of the denotations of its parts.

I know of no good evidence that Frege ever lost sight of the centrality of sentences--or propositional thoughts--in judgment, inference, language use–or in his method of reflecting on the semantical structure of language. He is clearly mindful of the deep differences between sentences and singular terms, even as he stresses semantical parallels. Frege's reasons for taking sentences to denote truth values are complex, pragmatic, and fully in accord with his fundamental philosophical aims. Frege's structural insight is compatible with his insight into the centrality of aiming at truth in language use. It is compatible with the centrality of reflecting on the preservation of truth in inference as the basis for understanding linguistic structure.

Part I of "Frege on Truth" traces considerations that went into Frege's treatment of truth

as the denotation of sentences. It also attempts to assess the relative weight and priority among these considerations. I argue that pragmatic and structural considerations dominate his thinking. This Part, which is quite complex, tries to work out details in Frege's method of investigating structure by considering function. He investigates logical and semantical structure by considering language use in relation to the function of language and judgment. Frege takes the central function of each as that of aiming at truth. He takes the central function of inference as that of aiming at preservation of truth. Part I is thus an introduction to the background thinking that led to Frege's conceptions of semantics and of logical form.

As I emphasize in Part I, Frege's basic considerations in reasoning about the denotation of sentences are structural and pragmatic. His view has, however, ontological implications. It is committed to taking truth, the truth-value truth, as an object. This view is widely and correctly regarded as unintuitive, and I do not accept it. Still, I try to show that the view flows fairly naturally from his pragmatic considerations–considerations about how best to account for the cognitively significant aspects of logic and language. Frege's ontological view is also deeply motivated by his belief in logicism. Part III of "Frege on Truth" attempts to elicit considerations that led him to take the truth value truth as the basic logical object.

With respect to these issues, "Frege on Extensions of Concepts, from 1884 to 1903" (1984), written earlier, should be read before "Frege on Truth". Relative to these issues, the two papers should be placed in the same Section in this collection of essays. The earlier paper also, however, broaches issues having to do with Frege's epistemology–his rationalism. I placed the paper in Section III of these essays, the section on epistemology, because I wanted to emphasize this aspect of the paper. But the issues of ontology and epistemology are closely intertwined. Here, as elsewhere, the reader would do well to read the essays in different orders and groups,

since any order or grouping has its limitations.

The problem of logical objects and the role of truth values in Frege's account is associated with at least three large, interconnected sources of philosophical interest. One lies in Frege's logicism. Frege's analysis of the structure of arithmetical thoughts and principles led him to believe that the numerals are singular terms. This led him to conclude that they denote objects. This view seems to me natural and probably sound. It raises difficult issues, however, for an account committed to logicism. For it requires not only that logic be committed to objects. It requires that it be committed to what are, to all appearances, particular objects. This commitment seems to be at least prima facie incompatible with the intuition, emphasized by Frege, that logic is completely general–hence not committed to any particular subject matter. It also raises a problem, raised by Kant: how, if arithmetic is a science whose basic laws are purely general, one can derive conclusions about objects like the numbers, with their particular properties–properties not shared by all objects.[13]

Some writers, both on their own account and as interpreters of Frege, hold that arithmetic is not committed to any particular objects: it is committed at most to structures that any of a number of different objects might fill. This is an interesting position. Discussion of it would take us too far afield. There is some textual basis for the position as an interpretation of Frege. But I believe that the preponderance of evidence counts against it. In my account, especially in Part III of "Frege on Truth", I follow the assumption that Frege regarded each number, like the truth value truth, to be a particular object. I think that Frege thought that there is a uniquely natural account of what the logical object truth (the truth value truth) is. He thought that the particular natural numbers are objects that can be explained in terms of this logical object.

I think that Frege hoped to reconcile this view with the generality of logic. He did so by

holding that since truth values are relevant to and in fact present in any subject matter, since the numbers can be explained in terms of the truth values as extensions of certain logical concepts, and since logical concepts are applicable in any science, arithmetic is a general science, not a special science. Numbers like truth values are implicit in any subject matter.

Frege's logicism failed. His thinking about what it is to be a general or special science and his reasoning about what sorts of commitments are universal to any science retain deep continuing interest. This is one of the ways in which Frege's odd-seeming view about truth-values as objects connects to independently interesting issues.

A second source of interest associated with logical objects and the role of truth values in Frege's account lies in his rejection of the idea that logic is true independently of any relation to a subject matter. This is the idea, mentioned earlier, that the positivists used to exempt logic from epistemic and metaphysical inquiry. Frege saw logic as the most general science of being. This is a traditional view of logic. It stems from Aristotle, saturates the medieval period, and runs through Leibniz, Bolzano, Frege, Russell, and Gödel. I think that it is clearly the dominant view in the history of logic. The view seems to me sound. Explaining in detail, however, exactly how logic relates to a subject matter, and how a subject enters into logical truths being true, is a complex and difficult philosophical matter. It seems to me a problem that remains a challenge worthy of serious attention.

Frege's approach to the problem is radical. He holds that every truth of logic is committed not only to logical functions (which he thought not to be objects) but to the truth value truth. That is, every truth of logic (indeed every truth) denotes that logical object. Thus truths of logic like all truths relate to the world through the simplest and most general semantical relation–the same relation that relates singular terms (or their counterparts in thought contents) to

objects. This idea seems to me profound in the way it capitalizes on the ubiquity of truth in framing an account of logic's ontology. Frege's forthright acceptance of the challenge to explain ways in which logic relates to the world also seems to me philosophically profound.

I do not accept Frege's radical approach. I think that the content of ordinary sentences is not correctly understood as a nominalization of the sentence, with a predicate like "is the true" applying to the nominalization.[14] So I do not believe that truth is under discussion, or is denoted, whenever assertions or judgments are made. All assertions and judgments presuppose a commitment to truth. But this commitment is not explicit in simple judgments. In order to make assertions or harbor beliefs individuals need not have the capacity to make it explicit. I believe that both developmental psychology and epistemology show that attributions of truth represent a cognitive step beyond the mere use of object-level sentences (or thoughts). I believe that these considerations support the view that predication is not strictly functional application, and truth is not strictly the topic or subject matter of all propositional contents–even the contents of assertions or judgments. Propositional contents have truth values. But I see no persuasive ground for reifying truth values, much less taking truth values to be the topics or subject matter of all propositional representation. Thus I believe that Frege made a mistake in his account of sentences as denoting truth values. But the mistake does not derive from failure to appreciate his own insight that sentences are deeply different from names.

There are two further grounds for not accepting Frege's radical approach. One is that it leaves one with unacceptably poor resources for understanding the semantical paradoxes, such as the liar paradox. The other is that his redundancy account of the sense of attributions of truth is cannot explicate the numerous contexts in which attributions of truth do not apply to fully specified sentences or thoughts.[15] Still, Frege's approach seems to me of deep philosophical

interest. It deserves much more reflection than standard dismissals have given it.

A third source of interest associated with logical objects and the role of truth values in Frege's account is the way they illustrate Frege's approach to the problem of how we can have apriori knowledge of a subject matter which we do not perceive and have no causal relations to. I will discuss this issue further in Section III. It is worth indicating here that Frege's account of truth-values as logical objects, discussed in Part III of "Frege on Truth", and his pragmatic investigation of the structure of language and logic, discussed in Part I of the same paper, are the roots of his approach to these issues.

I mentioned earlier that "Frege on Extensions of Concepts, from 1884 to 1903" (1984) might be seen as belonging in Section I of these essays, as a predecessor of "Frege on Truth"-- because of its discussion of Frege's account of structure and ontology. Here is a respect in which "Frege on Truth" should be read as part of the series of papers on Frege's epistemology.

The last paper in Section I, "Frege and the Hierarchy" (1979), deals with a more special issue in Frege's investigation of structure. It discusses the relation between Frege's theory of indirect denotation and his theory of sense. It attempts to explain principles that Frege was plausibly committed to, and which seem to me plausible in themselves, that generate his belief in a hierarchy of senses. He regards increasingly embedded indirect contexts as attributing senses increasingly high in the hierarchy. This early paper of mine is less explicitly historical than the others, but I believe that it does interpret Frege's actual position.

A number of significant philosophers, including Carnap, Dummett, and Davidson have found the hierarchy theoretically unattractive and even untenable. Church developed a version of Frege's hierarchical view. I believe that the hierarchy is not only tenable but nearly mandatory. I believe that alternatives for accounting for the relevant linguistic (and conceptual) phenomena

have serious liabilities. Understanding these matters is difficult, and may seem to be a rather specialized issue. But I believe that understanding the principles behind the hierarchy take one to the heart of not only Frege's point of view about sense and thought. The principles seem to me fundamental to understanding the functional character and productivity of sense and thought. I also believe that by understanding these principles one comes to have a deeper conception of what sense, as way-of-thinking or mode-of-presentation, is. I discuss these issues in a long postscript to the 1979 paper.

More broadly, simplicity, depth, and consistency of Frege's method of investigating structure, linguistic and conceptual, will provide any student of philosophy with valuable insights into language, truth, and thought.

Section II: Sense and Cognitive Value

Frege's notion of sense has undergone waves of critical opposition, from Russell, Wittgenstein, and the positivists, onward. Yet it or close analogs maintain a substantial presence in contemporary theorizing. I want to begin by mentioning two common types of opposition. Neither type seems to me to rest on solid grounds.

One type lies in expecting Frege's notion of sense to do explanatory work that it was not intended to do. There is a persistent tendency to see sense as a notion intended to explain ordinary linguistic meaning–the sort of meaning associated with communal mastery of the language. I shall be discussing this source of opposition in some detail in this section. So I will not say more about it here.

The other type lies in animus against the ontology of Fregean sense. This animus has a number of sources. I will discuss these briefly.

One source of animus is the assumption that Frege's notions of sense and thought content stand or fall with his Platonism about them. Few philosophers or linguists are comfortable with being as platonistic about any notion of representational content as Frege is. Although I have come to take Frege's Platonism about sense and thought content, at least about the senses of some logical and mathematical expressions, as more formidable than I used to, I myself do not accept it. I believe, however, that it is a very serious mistake to dismiss his conception of sense because one rejects this feature of it. Most of Frege's distinctive theorizing about sense and thought content is valuable independently of the Platonism. What is important is his identification of phenomena that need explanation, the form of explanation that he offers, and the structural and other explanatory insights that derive from his approach.

Many philosophers have maintained that it is a mistake to reify "meanings". This sort of rejection of Frege's notion is often associated with the conflation of sense with linguistic meaning, mentioned above. But it commonly rests on one of three other positions.

One is that there is some irresolvable problem about understanding the relation between speakers or thinkers, on one hand, and the senses or thought contents that they "grasp" or understand, on the other–if such senses or thought contents are "reified". A second is some form of nominalism about representational phenomena (both linguistic and mentalistic). A third is a generalized scepticism toward the objectivity of any discourse about such phenomena.

I cannot undertake to discuss this positions in any detail here. I want to signal my attitude toward them, however. I will remark on them in decreasing order of their ambitiousness.

Generalized scepticism about the objectivity of discourse about meaning, sense, and thought content is associated with Quine, and is sometimes echoed by Davidson. Scepticism is sometimes especially directed at such discourse inasmuch as it discusses sub-propositional

"intensional" structure. In my view, this scepticism has never been backed by credible argumentation. It has been ignored, and in my view bypassed, by the empirical development of linguistics and cognitive psychology. No good ground has been given for seeing these sciences as in principle less objective or less genuinely scientific than the natural sciences.

Nominalism about linguistic and representational phenomena was advocated by Ayer, Carnap, Ryle, Austin, Sellars, Goodman, as well as Quine and Davidson. Nominalism about linguistic and representational phenomena is, in my judgment, no better grounded than nominalism about mathematics. Explanation of linguistic and representational phenomena is enhanced by postulating abstractions, as is almost all explanation. To account for shareable linguistic and cognitive phenomena in a simple and uncluttered way, reference to and quantification over representational content that is abstract–not local to any particular place or time–is necessary. Relevant linguistic and cognitive phenomena are the compositionality of propositional understanding, the nature of reasoning and other psychological transformations, the relation between exercises of linguistic or psychological capacities and normative evaluations for veridicality and epistemic success.

I emphasize that not all commitment to abstraction constitutes commitment to Platonism. One might maintain more Aristotelian or conceptualist views about abstract entities. Getting right the exact ontology of the abstractions that are needed to explain linguistic and representational phenomena is complex and philosophically interesting. I believe, however, that it is a secondary matter. The key point about Fregean sense is its explanatory fruitfulness. I believe that one can see, rather easily, that such fruitfulness depends on commitment to abstract entities. But one can make considerable progress by making use of Frege's insights about sense and thought contents without deciding the exact metaphysical nature of the abstractions that one appeals to. This is the

analog of the situation with respect to mathematical entities in all the sciences. It is fairly obvious that one cannot avoid commitment to such entities or replace them with nominalized counterparts. The metaphysics of mathematical abstractions is, beyond these elementary points, of only tangential importance to explanation that makes use of them. I believe that the ontologies of mathematical abstractions and representational abstractions differ. But in neither case must philosophical and scientific progress await providing settled ontological accounts.

The view that there is an intractable problem about understanding the relation between speakers or thinkers, on one hand, and "reified" senses or thought contents, on the other, can be construed as an instance of a general problem about relations between individual thinkers and abstract objects. I believe that this problem is interesting and important. But I believe that our grounds to believe in the abstract entities are so firm that one can carry out mathematics and the empirical sciences with some confidence without having to solve it.[16]

Sometimes rejection of Frege's position depends on the idea that there is a special problem about the relation between individual speakers or thinkers and abstract entities, like representational contents, that the individuals "grasp". The question is how we can get "in touch" with Frege's thought contents (or senses) given that they are platonic entities. This source of ontological animus is more widespread than either of the other sources. It is shared by many philosophers who do not oppose abstract objects per se, and who are not sceptics about the objectivity of thought or meaning.

Here I believe that ontological animus against reifying "meanings" or thought contents depends on misconstruing the explanatory role of representational contents. I believe that their role is that of marking shareable states–whether these be linguistic or mental, and whether they be capacities, attitudes, or events. Representational contents play the role of helping to fix

explanatory kinds. Although the cases are different in several important ways, I believe there is no more difficulty in assuming a representational content associated with a word or a psychological state than there is in assuming a physical parameter or biological kind that has instances type-identified by that parameter or kind. In neither case is there a special problem about explaining the relation between property instances in time and the kind itself. Metaphysics can inquire into the relation. But science need not wait for the outcome of such inquiry. Fregean senses play the role of type-identifying linguistic usage, linguistic capacities (including understanding), psychological capacities, psychological states, and psychological events. They also play the role of type-identifying idealized justifications or argument structures. The reason why they should be taken seriously is that they contribute to explanation and to structural insight into the nature of linguistic and psychological phenomena.

Frege himself contributes to the concern that there is some special problem in explaining how we can grasp senses. He sometimes writes as if we have to explain how we can reach over some impossibly large divide:

> ...the law of gravitation...is completely independent of everything which takes place in my brain and of every change or alteration in my ideas. But grasping this law is nevertheless a mental process....in it something comes into account that is no longer mental in the proper sense: the thought; and perhaps this process is the most mysterious of all.[17]

Grasping a thought is simply a misleading metaphor. Any view should cash out the metaphor in terms of having a certain ability to think. Such an ability is attributable on the basis of ordinary evidence and is constitutively associated with a variety of applicational and inferential abilities. How does one "grasp" a thought content? One thinks it. Even on a Platonist view, the thought contents should be regarded as playing a role in type-individuating mental events, states, and abilities. On any view, the contents should be regarded as abstract, in order to account for

the multiplicity of instances of events that they type-individuate, to account for the shareability of kinds of thoughts, to account for compositionality, to account for the structure of inference and for various aspects of truth and justification.

Most formulations of the problem of explaining how we grasp an abstract entity, like Frege's formulation, present a pseudo-problem. Often the formulation takes Platonism to regard thought contents as objects that the individual thinker has to target or know about, or as entities that are "before the mind", in something like objects of vision. This amounts to a failure to maintain the distinction between sense and denotation. Such formulations have allowed the grasping or vision metaphors to distort what understanding is. They distort the explanatory role that thought contents play in type-identifying mental events, states, and abilities.[18]

For the Platonist, senses or thought contents can and should be regarded as constitutive elements in the individuation of capacities like understanding. The question how understanding can make contact with such entities does not make sense from such a point of view. Understanding is constitutively type-identified in terms of such entities in terms of thought contents. So the question of how a thinker or understander can "get in touch" with the entities should be, from the point of view of a serious Platonist, ill-formed. The problem of how a thinker can "grasp" the contents reduces to how a thinker can think thoughts with those contents. As is common in discussions of Platonism, a silly metaphor which is antithetical to the platonic point of view is substituted for the view; and then its silliness is presented as an objection. Problems with Platonism about senses or thought contents do not arise from its opening a chasm between "realms" which an individual thinker must somehow traverse. Problems arise from reflection on the individuation conditions for kinds of mental capacities. I shall return to these matters.

For now, I want to emphasize that the explanatory structure that Frege develops is more

important than his Platonism about representational content. Frege gives his notion of sense four main theoretical functions.[19]

First, sense enters into an account of thought and knowledge. Frege's first introductions of the notion of sense are tied to a notion of cognitive value–thought or knowledge potential. He also characterizes sense in terms of the notion of a mode of presentation–a way of being given to the mind. Senses of declarative sentences express thought contents. Structural parts of sentences express senses that express what are potentially components of thought contents. In theorizing about language, Frege's primary concentration is on a theory of thought and knowledge, as these are expressed through language. The workings of language, the details of linguistic usage, are of interest to him only insofar as they bear on how thought is associated with language in individuals or in communities of individuals. I will elaborate this central point further in this section.

It is important to bear in mind that Frege recognized that there are senses that do not determine a denotation or referent. This is to say that he recognized that thought does not infallibly connect to a subject matter. Names like "Odysseus" or "Ossian" may fail to denote anything. Definite descriptions formed from such predicates, like "the only round square", may also fail to denote anything. In the normal course of things, senses are ways in which entities determined by the senses are presented to the mind. All senses are representational and thus are essentially associated with a function of determining entities (denotations, objects of reference) presented to the mind. In this sense all senses "purport" to determine a denotation. They are purportedly presentations to a mind of a denotation. But they do not always realize this task or function successfully.[20]

Second, sense has a broadly semantical function. It fixes or determines a denotation. Moreover, if a sense determines a denotation, it determines a unique denotation. A sign denotes

its denotation and expresses its sense. The sense bears a relation to the denotation of the sign, the relation of determination, that is analogous to the sign's relation to its denotation, except that senses unlike signs cannot be ambiguous. In an ideal scientific language Frege held that signs are not ambiguous. They express exactly one sense. In an ideal scientific language, Frege thought signs would never lack a denotation. In such a language, signs would have exactly one denotation, and each sign's sense would successfully determine exactly one denotation.[21]

Frege took it to be a fundamental fact about thoughts--thought contents--that they can be true or false. It is clear that Frege does not regard being true or false as essential to being a thought. Thoughts containing components that are not associated with denotations are neither true nor false. I believe that he thought that successful denotation is the fundamental situation. Thoughts with truth value have an explanatory priority. But thoughts remain thoughts even when denotation fails and the thoughts are without truth value.

True and false thoughts are true or false eternally. Thoughts that lack truth value are also eternally truth-valueless. This is because thought components (including sub-propositional senses) denote or fail to denote timelessly. In fact, Frege holds that it is of the essence of thoughts that they are eternal–not in time (or space). Although individual thinkers think thoughts, the thoughts (thought contents) themselves are for him both abstract (not in space or time) and independent of any thinkers.[22] Since senses are essentially potential components of thought contents, they too are in themselves eternal, although they have the role of being senses only in relation to languages, which may well be thinker-dependent.

The claims about eternality are closely associated with Frege's quasi-semantical requirement of unique determination. Senses do not shift denotations with context. But the claims of the preceding paragraph also have a definite metaphysical caste in Frege's writing. Of

course, what sense is associated with what sign is dependent on individuals' mental and linguistic activity. In fact, whether a potential thought component is a sense depends on being expressed by a language. Languages may well be thinker-dependent. So something's counting as a sense may depend on thinkers. But the thought components that are senses (by virtue of being expressed by signs)–the entities that are senses--are themselves thinker-independent. Thought components and thought contents, which signs relate to when they express them as senses, are not to be explained in any way as dependent on thinkers or language users.

The third theoretical function of sense is to serve as denotation. We can denote the sense of a given sign S by using the expression "the sense of S". Or we can denote senses in attributions of propositional attitudes. Thus, for example, the sense of a singular term is, on Frege's view an object which can itself be denoted and thought about in attributions of propositional attitudes. An attribution like "Al believes that 2 is an even prime" uses a term–the that-clause–that denotes the thought or sense normally expressed by "2 is an even prime"; and the term "2" as it occurs in the that-clause denotes the sense that it would normally express if it were to occur outside of any that-clause. Frege provides an elegant and in its main lines plausible theory of how in serious psychological discourse, we can talk about thoughts–both the attitudes and the representational contents.

A fourth theoretical function of sense is that they serve in Frege's account of linguistic understanding. I would like to enter into an extended discussion of this function since its relation to the other functions–especially the first–is crucial to an accurate understanding of Frege's notion of sense.

Much of Frege's influence on thinking about language derives from his deep structural conceptions. A broader and more general sort of influence flowed from his conception of sense,

and hence of thought content, as objective and shareable. Frege's emphasis on these aspects of sense crystallized in two famous passages that deeply affected subsequent reflection on language. One passage occurs in section 26 of Foundations of Arithmetic (1884):

> ...we cannot even know whether [space] appears to one man as to another; for we cannot lay the spatial intuition of one next to that of another in order to compare them. But still there is contained in it something objective; all recognize the same geometrical axioms, if only through the deed, and one must do so to find his way around in the world. What is objective in it is the lawful, the conceivable, the judgeable, and what is expressible in words.[23]

The other occurs in Frege's most important and famous essay "On Sense and Bedeutung" (1892):

> A painter, a horseman, and a zoologist will probably connect very different ideas (Vorstellungen) with the name "Bucephalus". The idea is essentially distinguished from the sense of a sign in that the sense can be the common property of many and therefore is not a part or a mode of the individual mind (Einzelseele); for one can hardly deny that mankind has a common store of thoughts which are transmitted from one generation to another.[24]

In each of these passages Frege associates what is expressed in words with something shareable among different language users. Frege's emphasis on the shareability of sense derived from his interest in science. However, he applied it, with qualifications that will emerge, to ordinary use of natural language–as the "Bucephalus" example illustrates.

The positivists and Wittgenstein took over this emphasis. The positivists joined Frege in emphasizing the public character of activity that grounds meaning. They construed the activity that grounds meaning as confirmation procedures in science. The later Wittgenstein took up Frege's remark that recognition of objectivity or shareability in linguistic expression centered "in the deed". Wittgenstein came to criticize Frege's idea that different people might have different uncommunicable intuitions or ideas, a view that Frege developed further in the late essay "The Thought" (1918). But Wittgenstein's work clearly resonated to Frege's view that something

objective got expressed "in the deed"–in actual linguistic behavior and use in ordinary language.[25] In either application of Frege's point–to science or to ordinary language--the emphasis on publicity and shareability made investigation of language open to communal and scientific discussion.

The positivists' interest in the publicity of linguistic behavior and the shareability of linguistic meaning was motivated by concern to understand cognitive meaning in scientific enterprises. But the broader application of these ideas to all of natural language use emerged in the work of the later Wittgenstein, Quine, Strawson, and others in mid-twentieth century. This broader application came to be dominant in philosophy. This development in philosophy, along with the Frege-inspired approach to linguistic structure, nourished the emergence in mid-twentieth century of a science of structurally oriented linguistics for "ordinary" or natural language. Thus both Frege's concentration on scientific language and his application of his ideas to natural language contributed deeply to structural thinking about language and to the treatment of language as a public, communal enterprise.

In philosophy the emphasis on public, communal natural language went so far that an important aspect of Frege's thinking came to be obscured. The assumption that we speak a common language (e.g. English) and the assumption that Frege's notion of sense applies only to what is expressed in a common language, to conventional linguistic meaning, led many to the conclusion that Frege made a number of serious mistakes in his philosophy of language. These mistakes seemed to be magnified as attention turned to aspects of language that he and others thought should be excluded from the language of science. I have in mind context-dependent expressions, vague expressions, simple acts of communication not motivated by scientific concerns. In my view, most such lines of criticism of Frege are based on a systematic

misunderstanding of his notion of sense. Frege's conception of sense is deeply different from current conceptions of conventional linguistic meaning.

Frege's different conception of sense is signaled most clearly in his discussion of the senses of indexicals in "The Thought" (1918). There Frege completely ignores the obvious point that the conventional linguistic meanings of "I", "today", or a present tense construction remain constant from occasion to occasion and from speaker to speaker. He maintains that the sense (not just the denotation) of such expressions shifts from context to context. This discussion is clear evidence that Frege's notion of sense does not coincide with any ordinary notion of conventional linguistic meaning.[26] Frege's notion is essentially tied to thought. It is introduced primarily to serve his theory of knowledge. He is interested in language primarily as means of expressing thought. Communication, communal usage, communal understanding are secondary.

The evidence from Frege's treatment of indexicals is very strong and straightfoward. The main point of my paper "Sinning Against Frege" (1979) was to mobilize this evidence to support the contention that Frege's conception of sense differs from modern conceptions of conventional linguistic meaning. Frege's conception is concentrated on thought and motivated by a concern to understand knowledge. "Sinning Against Frege" was for me the beginning of an expanding realization of how markedly Frege's conception of sense differs from common conceptions of linguistic meaning in modern philosophy of language.[27]

Frege's orientation is quite explicit in his first introductions of the notion of sense, in "Function and Concept" (1891).[28] Frege argues first that the equations "2^4 [CHECK two to the fourth]= 4^2 [CHECK four squared]" and " 4 . 4 = 4^2 [CHECK four squared]" express different thoughts, though all the signs flanking the equality signs have the same denotation. All denote the number 16. He concludes from this that identity of thought expressed does not follow from

identity of component denotations. He argues for the same conclusion from the fact that "The Morning Star is a planet with a shorter period of revolution than the Earth" and "The Evening Star is a planet with a shorter period of revolution than the Earth" express different thoughts but have the same denotations. He means by this that an individual can use these sentences as vehicles for thinking different thoughts. He supports his claim that the sentences express different thoughts by noting that someone who does not know that the Morning Star is the Evening Star might accept the one thought and disbelieve or fail to accept the other. Assuming his composition principle that the truth value of a whole sentence is a function of the denotations of its parts, he supports his claim that the sentences have the same denotation (the same truth value) by noting that the sentences differ only through exchange of the words "the Morning Star" and the "the Evening Star". These words have the same denotation. Frege infers immediately from these grounds for distinguishing thoughts and denotations that we should distinguish sense and denotation. In fact, at the end of the paragraph he suggests that the sentences' having different senses just is "containing" different thoughts. This train of reasoning makes clear that Frege understands his technical notion of sense essentially in terms of its role in representing and accounting for what individuals can think when they use language as a vehicle of thought.

Similarly, in Frege's later but canonical and most famous introduction of the distinction between sense and denotation at the beginning of "On Sense and Bedeutung", he argues for differences in sense from differences in cognitive value (Erkenntniswerte), from differences in what is known, and from differences in epistemic status. Not very much later in the essay, Frege associates sense with mode of presentation (Art des Gegebenseins).[29]

Frege does write, three paragraphs further on, "The sense of a proper name is grasped by everyone who is sufficiently familiar with the language...". One must be careful about what Frege

means by "the language". The immediately succeeding passages make it clear that many people may grasp the same sense in a perfect or other scientific language. But he indicates that in natural languages an expression may not have a single sense, and that sense may vary with individual speaker and with context. In quite a lot of his work it is clear that Frege takes each person to have his own idiolect, commonly with idiosyncratic senses for proper names and demonstratives used in a context. So contextual ingenuity may be necessary to effect successful communication of thoughts.

Frege never clearly states that every sense must be graspable by others. He is, however, mainly concerned with shared or shareable senses.[30] He assumes that even in non-ideal non-scientific natural languages, expressions other than proper names, indexicals, and demonstratives have a common sense for most members of a community. So although he sometimes focuses on idiolects, he is also alive to the ways in which individuals share a language, or have significantly overlapping idiolects. It may well be true that for many expressions with a context-free communal meaning, Fregean sense will be identical with communal meaning.

The relation between communal languages, shared scientific languages, and individual idiolects in Frege's work is a complex one. He never concentrated on the topic. It is often clear, however, that he is concerned with the idiolects of individuals in his discussions of sense. This is particularly true when he is concerned with linguistic expressions that he would regard as not suitable for an ideal language for mathematics–expressions like ordinary proper names and indexicals or demonstratives.[31]

So when Frege states, "The sense of a proper name is grasped by everyone who is sufficiently familiar with the language...", he clearly envisions that "the language" can be either an idiolect, a communal natural language, or an ideal scientific language. If an individual expresses a

relatively idiosyncratic sense with a demonstrative or proper name, others may not be sufficiently familiar with "the language" to grasp the contextual or idiolectic sense. In the case of a communal language, the novice or the headstrong might well not grasp a sense shared by most others. In a scientific language those who are not properly trained may not grasp a standard sense. What counts as the relevant language cannot be assumed to be a shared communal language, or a name with a communally shared sense. Frege considers different types of language, of varying degrees of standardization, depending on the context of his discussion. What matters is what thoughts an individual or group of individuals use signs to think. The sense need not be stable across contexts, even in an individual's language.

I believe that Frege is completely consistent in using his notion of sense as a part of a theory of thought (or thought components) expressed through language. He is completely consistent in taking senses to be modes of presentation, the ways an entity is (actually or purportedly) presented to a mind when one is thinking through language. Language is necessary for the expression of thought. But thought is not, in general, to be explained in terms of ordinary conventional linguistic meaning.

Because of the immense inertia created by the concentration in twentieth century philosophy on language–more or less in abstraction from its relation to the thoughts of individuals–, Frege's notion of sense was commonly construed simply as communal linguistic meaning, as distinguished from reference or denotation. During the middle part of the twentieth century, in light of the widespread emphasis on the public, communal nature of language, communal usage and understanding in natural language came to be touchstones for assessing the notion of sense. Not surprisingly, Frege's conception of sense does not always happily explain matters that it was not intended to explain. Much of my work on sense has been an exploration of

the differences between Frege's conception and notions of meaning that were influenced by his conception but ultimately conflated with it.[32]

The evidence from the passages on indexicals that Frege held a thought-oriented conception of sense that differs from any modern conception of conventional linguistic meaning seems to me completely decisive and straightforward.

A rather less straightforward piece of evidence for the same conclusion lies in Frege's unqualified discussion in "On Sense and Bedeutung" (1892) of different speakers' "attaching" different senses to proper names, even names of individuals well known to the different speakers. Someone interested primarily in communal use and meaning would surely have qualified or explained these remarks. Frege simply takes it as obvious that, at least in natural language usage, different senses will be attached to the same name by different people, as different senses will be attached to the same indexical on different occasions by the same person.

Although Frege gives examples of the senses attached to proper names and indexicals, he never provides a systematic theory. It is likely that he gave no such theory because he thought that individuals could attach any number of senses to a name or indexical. Nothing in the language itself–either as communally spoken language or as a stable idiolect of an individual–constrains the senses that might be expressed by such expressions. That is to say that nothing in the language constrains very tightly the thoughts that might be thought through uses of sentences containing such expressions. Frege sees the indexicals as being recruited contextually by thought, as tangible but inexplicit and partial ways of expressing components of thoughts.

In 1970 Kripke and Donnellan presented powerful criticisms of "descriptivist" theories of proper names.[33] In my view, they established a number of important facts about the uses of proper names. These facts include the following three: a) The denotations or referents of proper

names are not fully fixed by descriptions that are associated with the names (either by individuals or in the community). b) Thoughts associated with the uses of sentences containing proper names are not fully accounted for by citing thoughts that are expressed by definite descriptions substituting for the names. Analogous points apply to indexicals and demonstratives. c) Causal relations--sometimes running through perception or perceptual memory, sometimes running through the passage of words from one individual to another--play a central role in combining with the individual's usage to fix the referent of names, indexicals, demonstratives, and indeed a variety of other expressions. This last point gained Kripke and Donnellan's initiative the somewhat misleading title "the causal theory of reference". Over three decades later there is still less a theory than a group of very generic principles and powerful reflective observations about particular cases.

Frege's examples of senses of indexicals and proper names all contain descriptions. In every case they also contain further names, indexicals, or demonstratives. Although Frege has been repeatedly criticized as a "descriptivist" about the sense (indeed the conventional linguistic meaning) of proper names, there is no evidence that he maintained any such general theory. That is, there is no evidence that he held a view that if the sense of proper name (on an occasion of use) were fully spelled out in language, it would in every case be fully expressed in terms of definite descriptions, with no admixture of names, indexicals, or demonstratives. I believe that Frege never thought the matter through in depth.

One way of defending Frege against charges of descriptivism is to appeal to "non-descriptive senses". I think that there probably are such things as non-descriptive senses–non-descriptive intentional thought components or cognitive values. I think that the senses of numerals (contrary to Frege's own view of the senses of numerals) may be examples. But for

reasons that I shall discuss in a moment, it seems to me that defenses of Frege along these lines are rather stretched and unconvincing.

There is another consideration that complicates criticisms of Frege from the point of view of the causal theory of reference. "Express" (as in "express a sense" or "express a thought") is a technical term for Frege. There is considerable textual evidence that he regarded the relation express between names and indexicals, on one hand, and senses (or thoughts), on the other, as very loose. Different people associate different senses with the same name or even with the same indexical in a given set of circumstances. Furthermore, there is no requirement in Frege's theory that when an individual uses a sentence containing a name or indexical on an occasion, the individual thinks only one thought. One might "attach" more than one thought to such a sentence on an occasion of use. As a substantive matter, it seems to me that this claim can hardly be doubted. So even if (as I believe) proper names almost never express definite descriptions, in the ordinary sense of "express" that characterizes common communal linguistic understanding, it does not follow that they almost never express definite descriptions in Frege's looser conception of "express". For it can hardly be denied that one can think thoughts partly expressible with definite descriptions meant to apply to the denotation of a name, indexical, or demonstrative, when one uses a sentence containing such expressions.

In view of the looseness of Frege's conception of "expression" when applied to names and indexicals (including demonstratives), one might offer against the standard criticisms the following defense. When one thinks thoughts in using such devices, one often thinks thoughts involving definite descriptions–although these will commonly include further names or indexicals. Who is to say that such thoughts are not among those expressed when one thinks thoughts through sentences containing names or indexicals? One might simply hold that Frege was not specially

interested in the sense of "express" in which it has been shown that names (and indexicals) hardly ever express complete definite descriptions.

I believe that any such defense of Frege would be partial, and ultimately insufficient to vindicate his views. It would be partial because one of the upshots of the points made by Kripke and Donnellan is that we think thoughts expressed by sentences containing names, indexicals, and demonstratives, that simply cannot be accounted for by appeal to complete definite descriptions. That is, thoughts involving some analog of the names or the indexicals have demonstrably different properties than any thoughts involving definite descriptions available to the individual. In numerous cases, it is simply not plausible that there are definite descriptions available to the individual that could fix the referents of those names, indexicals, or demonstratives. Frege did not seem to have been aware of this fact. I shall discuss it in more detail in a moment.

I believe it to be true that Frege was not interested in conventional linguistic meaning for indexicals and demonstratives. He did not seem interested in stating linguistic rules governing the conventionally understood use of such expressions. This seems completely obvious in his discussion of indexicals and demonstratives. So I think criticisms of Frege's philosophy of language regarding these devices–and even regarding proper names–are mostly not to the point. I believe that errors or oversights in Frege's view, in this area, reside primarily in his theory of thought, not his theory of language. His theory of language is aimed at accounting for different phenomena than modern theories are aimed at.

I will first discuss common types of criticism that rest on misunderstanding. Then I will discuss what I consider to be errors in Frege's views.

Let me go through a common dialectic with respect to Frege's best known example. Frege claims that "The Morning Star is identical with the Evening Star" requires a distinction

between sense and denotation. He states that the component expressions have the same denotations as the expressions in "The Morning Star is identical with the Morning Star", but the two sentences have different senses.[34] Frege reasons that since the senses of "is identical with" and "The Morning Star" remain the same between the two sentences, the senses of the two proper names must be different from one another.

A common objection is to claim that Frege made a mistake in holding that the two names have different senses. Such an objection often holds that names do not have sense or meaning. Sometimes Frege's reasons for his position are not noted at all. Sometimes it is held that Frege made a mistake in inferring from differences in knowledge–the difference between knowing that the morning star is identical with the morning star and knowing that the morning star is identical with the evening star–to differences in sense. It is held that the differences in knowledge do not reside in differences in the senses of component parts of the sentences.

These lines of objection ignore the fact that Frege understood sense in terms of cognitive value–in terms of contribution to potential knowledge. His notion of sense is introduced to explain differences of knowledge, not some independent set of phenomena. So it is mistaken to represent Frege as making a problematic inference from his point about knowledge to a point about sense. The inference is grounded in his notion of sense. This common line of objection does not attend to what Frege's notion of sense is. It assumes without argument a notion of sense that is not Frege's.

It is commonly claimed that names do not have sense because speakers of the communal language do not understand them in the same way. Or it is claimed that Frege made the mistake of taking a name to express the sense of a definite description: any name can be used so as to have distinct sense from any given definite description.

The first of these responses is a non-sequitur. Frege's notion of sense specifically does not require that names have a common sense in a communal language. Different individuals can express different senses. The senses are normally shareable, but they are commonly not shared. This view derives from Frege's primary concentration on cognitive value, not on communally shared meaning.

Evaluating the second claim is more complicated. In its usual form it depends on an assumption that Frege required senses to be (complete) definite descriptions. This claim that has no basis in the text.[35] Frege does give definite descriptions containing context-dependent devices as examples of senses "expressed" by names It is incontestable that such definite descriptions express thought components in any ordinary use of a proper name. Given the evident looseness of Frege's notion of sense-expression, it is hard to see these examples as incorporating a straight-forward mistake. I think that it would be a mistake to think that the thought content associated with a name is exhausted by such context-dependent descriptions. I believe that names are associated with thought content unlike any of the descriptive contents that Frege cites. Probably Frege failed to realize this. I will come back to this matter. The important point here, however, is that any commitment to holding that senses of names are those of definite descriptions is completely extrinsic to Frege's distinction between sense and denotation.

All Frege's argument depends on is the requirement that the name be associated on the occasion of use with some thought component (descriptive or not) that is distinct from the denotation of the name. I believe that this requirement cannot be circumvented. In understanding Frege, the important matter is to understand the notion of association or expression that holds between signs and thought components. Frege gives ample ground not to take this relation to be the same as that between signs and conventional linguistic meanings–shared by members of a

linguistic community. At least, in the case of names, indexicals, and demonstratives, it is very clear that Frege's remarks that senses are shareable are not to be taken to assimilate senses to communally shared linguistic meanings.

Let me take another example of a common misunderstanding of Frege's theory of sense. Frege elaborates his distinction between denotation and sense in such a way that a sense can be denoted as well as expressed. That is, one can use language to take sense as a subject matter, not merely a way of thinking about a subject matter. So the mode of presentation itself becomes the subject matter of a discourse. In some cases, this point seems completely obvious. I can use the expression "the sense of the word "brown" [or the sense of the numeral "729"] on occasion O". If O is a definite occasion, I can use the whole definite description to denote a certain sense.

Frege extends the point in an interesting way. He might ask us to consider the attribution, "Al believes that Mark Twain wrote Huckleberry Finn". Suppose that this attribution is true. Frege might then note that it is quite possible that "Al believes that Samuel Clemens wrote Huckleberry Finn" is false. Al might not know that Mark Twain is Samuel Clemens. Since "Mark Twain" has the same denotation as "Samuel Clemens", exchange of the two expressions in the original sentence ("Al believes that Mark Twain wrote Huckleberry Finn") should not change the truth value of whole sentence.[36] But there is a common understanding of the sentence according to which the truth value of the whole sentence does change under this exchange. Or better, the truth value of a thought commonly thought through use of the sentence does not change. So Frege concludes that the denotation of the two names is not the same in the context of the sentence and in the way they contribute to the truth value of similar sentences that contain them.

It is important to be clear about Frege's conclusion. Frege maintains (in effect) that in the sentence "Mark Twain is Samuel Clemens" the two names have the same denotation. Note that

exchange of the names in this sentence will not change the truth of the whole. But given his structural analysis that connects denotation with truth conditions, he concludes that in sentences that attribute propositional attitudes, the denotations of the two names can differ.[37]

Frege goes on to hold that in the context of the relevant sentences (understood in the way we have understood them), the names denote not their ordinary denotations but their ordinary senses. Similarly, the that-clauses in the relevant sentences denote not truth or falsity, but their ordinary senses–the thoughts normally expressed by those sentences. This constitutes a central example of the third of the four explanatory functions for sense that I listed earlier.

This view results in what I regard as a very beautiful structural account of the way that the denotation of sentences attributing propositional attitudes depends on the denotations of the parts of those sentences. What I want to focus on here, however, is that Frege takes such sentences to have components that denote the representational thought contents of the thinkers being discussed, including the components of those contents. The sentences are about Mark Twain via being about ways that Al thinks about him. These ways are denoted in the relevant sentences. When language is shared, these ways of thinking are the senses that the expressions within the that-clauses would ordinarily express if they occurred outside any that-clause, for example in a statement of identity.

Here again, a common body of criticism depends on misconstrual of Frege's notion of sense. It is often pointed out that there are readings of sentences like those about Al's belief that do allow for exchange of the names without changing the truth-value of the initial sentence. For example, the reporter might not care how Al thought of Mark Twain. So the report that Al thought that Samuel Clemens wrote Huckleberry Finn is just as true as the report that Al thought that Mark Twain wrote Huckleberry Finn. In a given reportorial context the latter attribution

might not only be true; it need not even be misleading. It is then argued that names lack sense because they are not associated with any commonly understood meaning (much less descriptive meaning). It is further argued that any construal of the sentence that depends on the vagaries of Al's idiosyncratic knowledge base cannot be shared in a communal understanding of the sentence. It is concluded that Frege made a mistake in holding that the sentences denote the ordinary senses of the relevant names.

Again, Frege's conception of sense is not that of communally understood linguistic meaning. The senses of such names are shareable, but only by understanding the ways that other individuals think about the names' denotations (or purported denotations). Frege places no requirement on sentences about propositional attitudes in ordinary discourse that the senses denoted be commonly understood or easily identified by others. Frege is also free to recognize the construals of relevant sentences in which senses of names within that-clauses are not denoted, although he does not discuss such sentences and may not have been aware of them.[38] He can account for such construals by holding that they rest on communicative purposes other than those of stating specifically and exactly the nature of an individual's attitudes.

Frege's theory applies to explicit and full specification of individuals' thoughts. It is not intended to capture common linguistic understanding. Nor need it apply to all uses of sentences attributing propositional attitudes. I think that there is no question that there is a place for language that is devoted to characterizing peoples' propositional attitudes as such. Finding the appropriate applications of the theory requires distinguishing sense from communally shared linguistic understanding of conventional meaning in a common language. Only in a regimented language devoted to full and explicit characterization of the way others' think–characterization of the specific natures of their propositional attitudes--would Frege's structural scheme for sentences

about propositional attitudes have full application. Frege's account is an account of thought expressed and discussed (denoted) in language–not a theory of shared communal usage or understanding.

I have mentioned that on Frege's view senses are "expressed" by linguistic signs, and are understood by competent speakers of "the language". I have entered significant caveats about the understanding of "expressed" and "the language". Although Frege ignored much that is of interest in the study of the regular use and meaning of natural language, I believe that if one bears firmly in mind exactly what he is trying to explain, most of the standard objections to his use of his notion of sense in his theorizing about language can be seen to be beside the point.

The basic problem with Frege's theory of sense resides in his theory of thought, not his theory of language. Imagine two cases. In one a child forms a perceptual belief. Because of a prismatic distortion he is misperceiving what is in fact a blue ball behind him, and believing of it that it is a red block in front of him. In the second case, a child remembers an object from perception of it. Suppose that she thinks a thought expressed with a demonstrative–"that green ball was fun to play with"; this time the child attributes "green ball" correctly to the remembered object. The child does not remember where the object was, or just when she perceived it.

In neither case does the child think meta-thoughts like the object I am perceiving or the object that I perceived. The child has not yet developed a capacity to think about his or her own perception. Even if he or she were to use such a description, the description need not be capable of picking out the object uniquely on its own. In the first case, the child might be unable to distinguish the time of the perception; the child relies ultimately on demonstratives like this moment or that body. In the second case, the child might well have perceived other green balls, and has no more than the correct attribution and the memory of the particular ball to single it out.

In each case, it appears completely clear that the child can think about the relevant particular object.[39] But no thought component available to the child that is context-independent (eternal in Frege's sense), and that marks a representational ability that is individuated independently of the particular context of application, suffices to determine the relevant object uniquely.

This is not a type a case that seems to have occurred to Frege. But how is he to account for the child's thoughts? He must postulate a singular sense or thought component, "expressed" by the term "that brown object", which uniquely determines the object. I accept this much. He must also take that sense to be eternal and independent of the thinker, indeed of any thinker. Is there some eternal determining element, some eternal thought component, that applies uniquely to that object, which type-identifies the child's thought?

It is hard to believe in such an element. Neither the children nor the objects are eternal. Each had a beginning and will have an end in time. How could some abstract, element of thought determine the object eternally and in a way that is completely independent of the child, and of the perceived or remembered object?

The problem does not lie in the abstractness of the element per se. The very idea of a cognitive value or a mode of presentation is one that, as Frege emphasizes, is naturally shareable. Shareability is naturally associated with some sort of abstraction from particular instances in individual minds. The problem lies, rather, in the independence of sense, or representational thought content, from thinkers. How is the thought component's intentional or representational character, the fact that it is part of a thought "of" the object, to be explained or explicated?

One can begin to imagine that for each object there are various ways to think about the object, fixed by the object's properties. But for any given property there are many ways to think

about it, ways that have potentially different cognitive values for a thinker. For example, different sense modalities seem to yield different ways of presenting a given property. Different angles of perception on the property within a given sense modality yield different ways. Different capacities within a sense modality (for example, whether a color is represented in a color-blind way or in a normally colored way or in a way affected by different levels of color-sightedness) can be associated with different cognitive values. A thinker may find informative a thought that constitutes an identification in which one representation of the property is taken to represent the same property as the other. Similarly, for informative thoughts about the identity of objects, or any other type of subject matter. It is hard to see how all of these perspectival modes of presentation could be individuated independently of actual minds and actual objects that individuals interact with.

The problem lies not only in the multiplicity of representational types and their seeming parochialness, their dependence on the perceptual and conceptual idiosyncracies of actual individuals or actual species. The problem lies even more fundamentally in the fact that there can be different instances of a given property specified by a given representational type. What instance is perceived, remembered, or thought about commonly depends on the particular contextual (often causal) relations that the thinker bears to that particular. General representational abilities are often not refined enough to single out the particular by themselves.[40]

Frege can consistently postulate eternal thinker-independent thought components to correspond to each of these different ways. But such postulation breaks with credibility. Let me summarize what seem to me to be separable but closely related difficulties with Frege's theory of thought.

In the first place, the particular content of many thought components makes implausible

his apparently generalized commitment to the view that thought components have being independently of thinkers. Many thought components seem to be contingent on either the nature of thinkers or the nature of contingently existing temporal entities. It appears obvious, for example, that different degrees of color sightedness or different types of sense modality are individuated and explained in terms of the mental capacities of thinkers. Modes of presentation in thought can depend on such matters. Similarly, the idea that a conceptual thought component such as that of a harpsichord or a dodo has its being independently of any relation of thinkers to historically contingent individuals of the relevant kinds seems very hard to accept. There seems no strong reason to accept it.

In the second place, Frege's view that the representational characteristics of such thought components are, in general, what they are independently of thinkers seems to me to be a piece of magic thinking. It appears obvious that the representational or referential characteristics of such thought components are to be explained in terms of thinkers' activities or capacities–in terms of actual and potential responses to causal interactions with relevant objects. This point seems to me to apply both to thought components representing repeatable types–such as properties or kinds–and to thought components representing particulars, such as the particular object that the child remembers.

In the third place, Frege's commitment to the view that thought components determine their denotations independently of any particular occasion of thinking–any particular contextual relation between the thought and the denoted entity--fails to be psychologically plausible for numerous cases involving names, indexicals, and demonstratives. The problem is especially acute for names and for demonstratives grounded in perception and perceptual memory. Let me try to make the problem vivid by contrasting the context-dependent thought components with occasion-

independent thought components. Let us take as an example of thought content containing only occasion-independent thought components all beagles are mammals. The thought component mammals represents mammals independently of any particular occasion of thinking with that thought component. Such thought components mark repeatable abilities by a thinker capable of thinking thoughts containing it. The thinker must have causal relations to something (I think usually but not necessarily mammals) in order to represent mammals. But any of various ways of acquiring the concept will do. The ability to represent mammals is repeatable in the sense that it is not essentially tied for its individuation to any particular token interactions with the world.

Frege and many neo-Fregean theories that appeal to "non-descriptive senses" apply his doctrine of occasion-independent determination of represented entities across the board–to every thought component representing every entity. They are committed to maintaining that thought components are fundamentally types, whose identity and representational characteristics are independent of particular events in space or time. So the thought component that determines the object remembered by the child marks a repeatable ability by the child to represent the object–a repeatable ability whose determination of the object is not essentially dependent for its individuation on any particular occasion of thinking about the object. But it is clear that the child's thinking about that particular object can be explained only by reference to the child's causal relations through memory to that particular object. It can be explained only by reference to particular acts of thinking and particular causal relations to that particular object. Generalized repeatable abilities are not sufficient. The child has no way of determining that object independently of particular occasions in which she has been in causal relations (through perception) to it. The representational ability is essentially tied to particular token acts and particular causal relations. I believe that this third point is the most fundamental one.

Some aspects of thought, particularly empirical thought, are irredeemably context-dependent. They are essentially explained in terms of certain particular acts and certain temporally specific causal interactions. Given the limited powers of actual thinkers, some thought about particulars depends irreducibly for its representational identity on certain actual, contextual relations to certain particulars–relations not explainable in terms of general abilities of those thinkers, or context-independent thought components that mark such abilities. I believe that our understanding of thoughts expressed with proper names, indexicals, and demonstratives frequently force these conclusions. This point seems to me to constitute a sound criticism of Frege that derives from the work of Kripke and Donnellan on singular linguistic reference. More generally, the explanatory dependence of thought on thinkers' relations to kinds or types of entities–discussed in the first two problems for Frege's view--runs counter to Frege's platonic position that all thought contents and thought components are thinker-independent.[41]

The problems that I have just discussed are most acute for thought about contingent objects through empirical means. It would be easy for a mathematician to fall into a platonic view of all thought content. A relaxed ontological Platonism about mathematical entities is a comfortable and plausible position for a mathematician. It would be easy to transfer Platonism from mathematical entities to mathematical thought contents and then to empirical thought contents, if one is not specifically focused on empirical thought. Platonism about the contents of empirical thought is much less plausible than Platonism about mathematical or logical objects or functions. Problems of the sort just discussed are also less easy to press for mathematical and logical contents of thought than they are for empirical thought contents. I remain sceptical about Platonism regarding thought contents, even logical and mathematical thought contents. But the issues are more complex in these areas. I shall not pursue them here.

So far I have distinguished sense from shared conventional linguistic meaning. I have associated sense with a theory of thought through language not communal linguistic usage. I now want to broach an aspect of Frege's theory of thought that makes his notion of sense even more deeply different from most modern conceptions of meaning.

Frege held that what sense an individual's sign expresses is not fixed by communal linguistic usage and understanding. There is strong textual evidence for thinking that he went much further. It is, in a certain way, not fully fixed either by the individual's own usage and understanding or by use and understanding present anywhere in the individual's linguistic community. This is not a point about the ontology of the entities that are senses. The point is about the relation between linguistic signs and usage, on one hand, and the senses or thought contents expressed by them, on the other. The connection between individuals' expressions (or thought events) and the contents that are thought does depend, at least partly, on the individual's activities and capacities. Individuals express senses and think thoughts only by having certain linguistic and mental abilities, and only by doing certain things. The question is how to understand the relation between these abilities and activities and the expressions of senses or the thinkings of thought contents.

The question can easily invite reduction. Many of Frege's successors attempted to reduce meaning to confirmation procedure, or "use", or functional states defined on the proximal inputs and outputs of individuals. Until the last three decades of the twentieth-century, even those who did not propose reductions believed that what "meaning" is expressed by an individual's words is fixed or determined purely by the individual's explicatory abilities and linguistic understanding, fairly narrowly circumscribed.[42]

Frege does not focus on the question. By later standards – associated with attempts to

give theories of meaning or explain what meaning "consists in" – Frege has too little to say about understanding and about what determines what sense a word expresses. This is partly a product of his taking sense and denotation as theoretical relations that do not need reduction or philosophical explication so much as fruitful theoretical employment. But there is no question that in light of later concerns, Frege seems relatively uninterested in or oblivious to philosophical issues that cluster around the question. Nevertheless, I believe that Frege had views on the question that are extremely interesting. In some applications, his views are exciting, fruitful, and plausible.

There are a number of places where Frege claims or presupposes that individuals can express senses, or think thoughts through the use of language, that they do not have the background knowledge to fully understand. Such understanding may be lacking even in the individual's broader linguistic community. Frege had particularly in mind cases in mathematics. A relatively vivid example is Newton's thoughts about limits in the calculus. The term "limit" did not receive a fully coherent and adequate mathematical explication until two centuries after its introduction. Frege would have regarded it as having a constant sense and denotation from the time of its initial seventeenth-century uses by Newton. But it came to be fully understood only with the definitions arrived at in the nineteenth century.

The central case for Frege was the term "natural number" itself. Frege thought that no one had adequately understood the term until his logicist explication of it. There is good textual reason for thinking that he thought that the sense of the term "natural number" (or its German counterpart) had remained constant, but that mathematicians had had only an incomplete and infirm understanding of that sense. It is evident that he thought that only with the development of an adequate understanding of "function" and only with the discovery of the functional character of

predication, was an adequate understanding of the sense of "natural number" possible.

It is hard to overemphasize how different, in this respect, Frege's conception of sense-expression is from nearly all conceptions of linguistic meaning (or indeed the contents of thoughts) that were developed in the wake of Frege's work. Nearly all other conceptions sought to ground meaning entirely in some sort of actual pattern of use or some sort of actual articulateable understanding by the individual, or individuals, whose expressions have the meaning. It is natural to think that either the individual or at least the community fixes the meaning through an expression's use or understanding.

There is indeed a fruitful conception of meaning that follows these natural thoughts. Frege's conception of sense, however, is different. I believe that it is also a fruitful. I believe that it points at a genuine and important aspect of the nature of thought and linguistic expression.

Earlier, I criticized Frege's conception of sense because of its extreme idealization. I held that its focus on mathematics leads to neglect of particularistic, context-dependent aspects of thought. Here, I believe, we find a respect in which Frege's extremely idealized conception of sense, governed by his extremely idealized conception of an ideal scientific language, yields insights.

Frege believed that norms of reason play a role in determining the nature of an individual's thought and the sense of an individual's (and community's) linguistic expression. He believed that given the function of mathematical thinking and given the fact that mathematical thinking is basically on the right track, the senses of mathematical expressions are partly determined by their role in a correct and uniquely appropriate rational elaboration of actual usage and of incomplete understanding. Thus he regarded his logicist elaboration of ordinary arithmetical discourse as revealing what traditional mathematicians had been thinking with incomplete understanding. It

revealed the senses that arithmetical expressions had been expressing all along, even though actual usage and understanding in the past was insufficient in themselves to fix the sense of the expressions. He thought that this elaboration also revealed, of course, the nature of the entities, the numbers, that had been thought and talked about.

In the abstract, this view may seem somewhat far-fetched to those more used to ordinary conceptions of communal or idiolectic linguistic meaning. The failure of Frege's logicism makes this particular application of the view unappealing. But in fact, the view corresponds to something deep about the nature of thoughts. In both the history of natural science and the history of mathematics, there is a strong pull to attributing a conception to individuals who get on to the basic features of a subject matter, even though they have not fully mastered the conception. The example of Newton on limit is one such case. The examples of Newton on mass or (mathematical) function, Leibniz or Descartes on inertia, Dalton on atom, Mendel on gene, Abaelard or Bolzano on formal consequence, are others.

Discussion of this aspect of Frege's work is first broached in Part VI of "Frege on Extensions of Concepts, From 1884 to 1903" (1984). It is more fully articulated and developed in "Frege on Sense and Linguistic Meaning" (1990). The later paper teases Frege's view out from discussion of a puzzle about his views on vagueness. Once one is sensitized to its presence in his writings, it is easy to see it in quite a lot that he writes. I believe that this discovery about Frege's view is historically important because it provides a much richer background for his conception of sense as fundamentally part of a theory of thought and knowledge. I regard the account of this aspect of Frege's work as philosophically the most important discovery that I have made in working on Frege. I see "Frege on Sense and Linguistic Meaning" as philosophically the most significant of my Frege papers, although I do not regard it as being as historically rich as some of

the others.

This view of Frege's is philosophically important, in my judgment, because it makes an important contribution to anti-individualism about the individuation of mental states and about a kind of linguistic meaning-expression. Anti-individualism is the view that the individuation of certain mental states–the explication of necessary conditions on what it is to be in certain representational mental states–necessarily involves relations between the individuals in those states and aspects of an environment which is the subject matter of those states.[43] Anti-individualism is an old view in philosophy, going back at least to Aristotle. The view has been more deeply developed in the last quarter-century through its alliance with developments in the theory of linguistic reference (cf. note 33.)

Frege's view is a form of anti-individualism. He holds that an expression can express a definite sense even though the individual's relation to that sense is not entirely explicable in terms of what the individual does with the expression–how he uses it or how he would explicate it. The explication of what sense is expressed is supplemented by reference to rationally understandable aspects of reality underlying mathematics and other sciences–aspects that no one, including the individual who expresses the sense, has yet fully understood. This sort of explication is a type of individuation. In other words, Frege holds that sense-expression–what sense a linguistic sign expresses–is individuated partly in terms of the individual's relations to a reality beyond the individual. Frege understands sense in terms of mode of presentation, and sense-expression in terms individual's mental states in the use of language. So individuation of sense-expression is individuation of mental states. So Frege's theory of both sense-expression and his theory of mental states is implicitly but clearly anti-individualistic.

My discovery of Frege's application of anti-individualist views to mathematics and his

understanding of sense within a rationalist framework made an important contribution to my thinking about meaning, mind, and knowledge. It enriched the development of anti-individualism, which had already independently become prominent in philosophical discussion. It also enriched the development of rationalism, a topic I will discuss shortly.

Frege's rationalist anti-individualism is congenial with his ontological Platonism about senses–the view that senses themselves are independent for their natures from thinkers. Frege's rationalism is incompatible with a traditional epistemological Platonism, or at least the traditional caricature of epistemological Platonism. According to this traditional view, understanding is construed in terms of immediate insight, and glossed in terms of the vision metaphor. In mathematics, if what content a thinker thinks or what sense his word expresses is dependent for its individuation on a thinker-independent subject matter of numbers and functions, it is easy and simple to treat the senses as themselves thinker-independent. Then the activity of thinkers plays an individuative role only with respect to what content is thought and to what sense is expressed. It plays no role in individuating the nature of the contents or senses in themselves.

But Frege's views about individuation of sense-expression do not entail ontological Platonism about thought contents or senses. I believe that these views are best developed in such a way as not to accept a generalized ontological Platonism about senses. Senses are appropriately seen as abstract. But their natures–like the natures of concepts (in the traditional, non-Fregean sense of this term)--are, at least in most cases, better regarded in an Aristotelian or a conceptualist way. It is hard to see a perceptual concept of a beagle, or a concept of Beethoven's Harp quartet as independent for its nature of any activity by any thinker. Their abstract identities are not independent of patterns of activity by thinkers in time.

It seems to me that philosophy of language is even now so focused on conventional

linguistic meaning that it has not adequately exploited Frege's notion of sense. In philosophy the idea that meaning is a common denominator shared among normally competent speakers of a communal language continues to dominate reflection on language. This focus has yielded many insights, and Frege's role in establishing the focus is fundamental. But I believe that taking better account of Frege's notion of sense will enrich theoretical understanding of language. It opens a role for the normative aspects of use, takes account of the complex relation between language and individual thought "expressed" in language, and highlights the historical role of the subject-matter of a discourse in determining the contents of thought and the sense of linguistic expression.

Linguists in their focus on idiolects rather than communal languages have tended to be more attuned to psychological underpinnings of language. But they have not, in my view, explored linguistic idiosyncracy very far, much less exploited the anti-individualist and rationalist aspects of Frege's conception. It seems to me that Frege's conception of sense remains a potential source of insight for both philosophers and linguists. I shall return to it in discussing his rationalism.

Section III: Rationalism

Frege's conceptions of sense, sense-expression, and sense-understanding must be understood in the context of his larger rationalist views about knowledge. The last group of papers center on Frege's epistemology. Rationalism is the view that some human knowledge has justification or warrant that does not depend for its justifying force on sense-experience. Knowledge not warranted by sense-experience is usually taken to include logic, pure mathematics, and parts of philosophy. This negative characterization is basic. More positively, rationalism usually maintains that understanding, reflection, or reason suffices to support certain

types of knowledge.

Frege was a thoroughgoing rationalist. In some important ways, he was a traditional rationalist. He held that knowledge of logic and of the mathematics of number is warranted through reason and understanding. He held that basic principles of these disciplines are self-evident. He thought of them as self-evident not in the sense that they are obvious, but in the sense that they yield in themselves evidence for believing them, if one adequately understands them. Frege was also traditional in believing that there is a natural order of priority among truths in mathematics and logic. This order corresponds to the structure of proof. Frege regarded proof as not purely a matter of establishing what follows from what, but as a matter of justifying a conclusion. This is a traditional view of proof. He thought that more basic truths provide the most fundamental reasons for believing derivative truths. Thus he thought that logic and the mathematics of number rest on a foundation of truths that do not need further justification, because their content provides sufficient "evidence" or justification for belief in them, indeed for knowledge. He held that such foundational truths do not admit of further justification, at least not further ideal or canonical justification. They do not admit justification because their self-evidence is sufficient in itself for knowledge and belief, and not subject to justificational improvement.

With respect to geometry Frege was also a rationalist. But in this case he maintained a more complex, purportedly Kantian position. He held that knowledge of Euclidean geometry is warranted through a combination of understanding and pure intuition–a capacity to intuit aspects of space and time without direct use of the senses. Frege applied his traditional views about self-evidence and justificational priority to geometry. His main focus was, of course, on knowledge of logic and the mathematics of number.

Frege was aware of the existence of non-Euclidean geometries. He thought of them as

mathematical curiosities. He recognized that they are consistent. He maintained that pure intuition shows them to be false.

This position now seems outmoded and difficult to justify. Mathematical practice has validated the non-Euclidean geometries. The notion of pure intuition has remained somewhat obscure. No powerful case has been made that any such capacity shows that non-Euclidean geometries lack mathematical truth. The non-Euclidean geometries gained status through serious arguments that one of them applies to physical space more accurately than Euclidean geometry. Frege could not have foreseen these developments. Still, they tend to undermine his view about non-Euclidean geometries and even his view about the role of pure intuition in supporting any of the geometries. A natural view now, which I believe that Frege would have embraced had he known more, is that at most one of the geometries best applies to physical space; but as pure mathematics, the main geometries are not incompatible. They describe different but genuine geometrical structures. On this view, as pure mathematics, both Euclidean and non-Euclidean geometries are true.

Whether Frege would have held to his view that some type of apriori intuition–some apriori capacity for singular representation grounded pure geometries–is a matter of conjecture. Whether we should regard pure geometry has depending on some such abstract form of singular space-like representation seems to me a question well worth investigation. What is clear is that he was mistaken in thinking that Euclidean geometry can be known apriori to be applicable to physical space, and in thinking that it is the only geometry, as pure geometry, that is true.

Frege's error about the non-Euclidean geometries was commonly associated by logical positivists, and others among Frege's successors, with the spectacular failure of Frege's own Logical Law V. Frege proposed this principle as an axiom in order to account for what he

regarded as logic's commitment to objects–principally the numbers. The principle maintained a systematic association of extensions with predication. As is well-known, Frege's principle was shown by Russell in 1902 to lead directly to contradiction. Russell's demonstration–now known as Russell's paradox–defeated Frege's attempt to demonstrate the truth of logicism. Although it contributed to an explosion of attempts either to establish logicism by another route, not least by Russell himself, or to develop a non-logical, set-theoretic foundation for mathematics, it effectively ended Frege's intense commitment to investigating the foundations of arithmetic.

Russell's paradox and the non-Euclidean geometries were long held to have an even more pervasive effect on Frege's philosophy. Logical positivists and others used these examples as bases for claiming that Frege's rationalism, indeed any rationalism, is untenable. The idea was that the two cases showed that rationalist appeals to knowledge warranted independently of experience are completely unreliable.

Not all of Frege's successors accepted such a view. Russell and Gödel did not. But the examples had the effect of giving rationalism a bad name. Euclidean geometry had long been the rationalists' prime example of a case of apriori knowledge–indeed apriori knowledge of physical space. Frege's Logical Law V was often presented by his successors as a principle that seemed perfectly obvious but which turned out not to be true at all. The positivists tended to see the different geometries as competing empirical theories of physical space. As we have seen, they treated logic and pure mathematics as degenerately true–as not about a subject matter at all. Even after the demise of logical positivism in mid-twentieth-century, different versions of empiricism hung on. As I have mentioned, it was not until late in the century that empiricism came to be re-examined; and rationalism was taken seriously again.

I believe that the history of geometry reveals a much more complex story than the simple

one presented by Frege's successors. The key Parallel Postulate of Euclidean geometry was long regarded by mathematicians and informed rationalists as less obvious than the other principles.

Unlike the Parallel Postulate, Frege's Logical Law V is simply false. It did not undergo or withstand scrutiny for nearly as long as the Parallel Postulate did. The historical story associated with it is, however, again much more complex than the one popularized by Frege's successors. In "Frege on Extensions of Concepts, from 1884 to 1903" (1984) I trace the development of Frege's thinking about the key notion, extension of a concept, and about Logical Law V. I show that Frege did not take the law to be obvious. He struggled through a complex web of considerations, which even seemed to include attempts to avoid reliance on the law--probably because of dissatisfaction with his understanding of the key notion.

This paper is a case study in Frege's particular form of rationalism. What it shows is that despite Frege's claim that basic logical laws are "self-evident", he did not regard all basic logical laws as immediately obvious to a reflective mind. He did not regard self-evidence as subjective or psychological obviousness. He took logical laws to be objectively self-evident and to be subjectively obvious only to a mind that adequately understands them. Like any traditional rationalist, he left room for incomplete understanding that might well lead to mistakes about whether a proposition is a logical law--and even whether it is true.

Most traditional rationalists were fallibilists in this way. They held that purportedly apriori justified beliefs might be mistaken. But traditional rationalist epistemology nevertheless relied on immediate insight. Many rationalists made claims to knowledge with no more justification than an appeal to self-evidence, in both objective and subjective senses of the term. And often these claims did not withstand the passage of time. Philosophy is, of course, strewn with such claims--later given up, ignored, or seen as mere historical oddities. Frege's empiricist successors

often derided the traditional rationalist belief in incomplete understanding as a mere safety valve. They saw it as a kind of dodge that rationalists appealed to, to avoid embarrassment, in case a given claim turned out to have objections to it.[44] The appeal to incomplete understanding was out of step with accounts of meaning in terms of use or confirmation procedures.

The idea that mind is to be partly explained in terms of rational structures that are more fundamental than individual minds is also a traditional rationalist view. This view was largely ignored until very late in the twentieth century. It too was out of step with accounts of meaning in terms of use or confirmation procedures. It took the revival of anti-individualism to make the view seem relevant to contemporary thinking.

In retrospect, the traditional aspects of Frege's rationalism take on different significance in his work in the light of the original elements in his rationalism. There are two broad aspects to Frege's thinking about these matters that are, I think, new in the history of philosophy, and that constitute important contributions. One is, again, his systematic and fruitful connection of issues about mind and knowledge with issues about logic and language. Frege's conception of sense is extremely original in the way it is integrated into a systematic theory of language and thought. His systematic approach to the understanding of thought through understanding language and deductive inference offers the same advantages here as it does elsewhere in his philosophy.

In particular, Frege's holist method for investigating logical form is simultaneously a method for finding and clarifying logical law. So full reflective understanding a logical law is not adequately characterized simply as immediate recognition of the obvious. Any such recognition is the product of a background of implicit reflection on inferential practice and structure. Similarly, intuitive understanding of a logical law presupposes an intuitive but discursive competence in carrying out deductive inference. Frege's reflective method clearly depends on the development

of theory–not on putative simple, immediate insight. The method presupposes that even unreflective intuitive understanding is ineliminably entangled with a complex web of standing inferential capacities.

Frege's method also allows a wide-range of considerations to affect the form and substance of theory–considerations about the functions of arithmetical discourse in counting for example, or considerations about simplification and unification of different aspects of his logic. Frege's epistemic practice is fundamentally pragmatic. It takes understanding to involve holistic, inferential elements. It is not an epistemology of groundless dogma or of immediate insight.

The other new aspect is his association of his rationalist conception of understanding with the history of science. Major rationalist philosophy before Frege was relatively ahistorical. Frege's historical perspective on the development of logic and mathematics gave him concrete illustrations of incomplete understanding. Frege had the benefit of reflecting on how poorly basic mathematical notions in the calculus had been understood and how they eventually found precise, even definitional, explications. He could rightly regard his development of logic as the clarification of incompletely understood logical concepts and structures. He saw his understanding of number as the culmination of a historical development–attaining a new understanding of a term that had carried its sense through harder more ignorant times. These cases were not used as dodges. Frege used them to illustrate how incomplete understanding could be clarified and made firm.

Frege said only a little about natural science. What he did say, together with his views about the history of logic and mathematics, makes Frege's conception of sense and his broad emphasis on the possibility of incomplete understanding relevant to understanding the history of empirical science as well as mathematics. Often it seems fruitful and correct to interpret language

and thought in natural science in terms of a concept or meaning that denotes a kind before true scientific understanding of the nature of the kind had been achieved. This is true of Dalton's discovery of the atom. Even while he mistakenly defined "atom" as the most basic, indivisible particle, he had a grip on a term that denotes the natural kind. I know of no reason to think that the sense of his term "atom" was different from ours. Here it appears that the denotation is fixed by relations of the scientific discoverers to the actual kind, and the concept or sense expressed is fixed partly through the denotation, partly by approximation to a correct theory or by a fruitful experimental method.[45]

The two new aspects of Frege's rationalism–logical/linguistic and historical–combined to yield another significant contribution. As I have intimated, he completely changed the traditional conception of understanding. Frege's pragmatic approach to understanding linguistic structure, embodying what I called earlier a fundamentally synthetic method, led him to alter the traditional rationalist conception of reflective, explicative understanding as simple insight. Frege saw such understanding as inextricably associated with systematic, theoretical reflection on a variety of inferences within substantive, scientific theory. So understanding is discursive rather than quasi-perceptual. It is essentially connected to inference. The metaphor of immediate insight is, from his perspective, misleading. What is fundamental for understanding–and ultimately for many rationalist grounds for belief–is holistic connection of inferences. Where understanding provides apriori grounds for belief, it is associated with holistic inferences within an apriori theory, or apriori aspects of theory.

Frege's historical perspective on the changing understanding of terms in mathematics and empirical science, and especially on the historical development of logic culminating in his own discoveries, led him to construe understanding not as a matter of simply analyzing meanings that

were already "implicitly understood". Understanding a sense or a thought is inextricably associated with substantive knowledge. In fact, in some cases, gaining a fuller understanding of one's own thoughts is inextricably associated with acquiring new substantive knowledge about other matters. The conditions that individuate concepts or sense-expression lie partly in substantive matters that may not be available to simple reflection, unaided by scientific discovery, no matter how sophisticated the reflection may be. Thus Frege's development of the notion of sense carries him very far not only from traditional rationalism–the rationalism of intellectual vision or insight on brief reflection. It also carries him very far from empiricist construals of understanding in terms of unpacking of meaning already implicit in use. It places him even further from the positivist conception of understanding of meaning as "analytically" independent of knowledge of a subject matter.

Frege's philosophical views, and especially his practice as philosopher and mathematician, give fuller substance to rationalist fallibilism and to rationalist appeals to incomplete understanding. I have highlighted Frege's historical perspective and his pragmatic holist method for investigating logical form as key elements in his originality. These elements enabled him to cast rationalism in a new light. Frege's rationalism was quietly embodied in the work of Church through much of the century. Rediscovery of its role in shaping his conception of sense and in providing perspective for his conception of incomplete understanding helped inject life into the re-evaluation of rationalism that has been proceeding over the last two or three decades.

"Frege on Knowing the Foundation" (1998) consists of a detailed examination of both sides of Frege's rationalism–the traditional aspects of it and the more pragmatic and historical aspects. The paper centers on the meaning of Frege's notion of self-evidence, and on his particular treatment of axioms in his logic.

It seems to me that Frege's rationalism is not vulnerable to criticisms usually put to it by his successors. In the first place, scrutiny of the relevant history indicates that the key examples can reasonably be attributed to incomplete understanding. There was a history of worrying about whether the relevant principles (the Parallel Postulate and Logical Law V) were adequately understood, before mathematical developments showed their true status. In the second place, Frege's epistemology does not sanction dogmatic invocation based merely on claims of insight. In the third, there is no good ground for holding that Frege's method of investigation is unreliable or that ordinary non-empirical methods in logic or mathematics are unreliable.[46]

The pragmatic, discursive aspects of Frege's rationalism seem to me to cast Frege's Platonism in an attractive light. "Frege on Knowing the Third Realm" (1992) discusses Frege's ontological Platonism and his view of our knowledge of entities that are not in space or time, to which we bear no causal relations, and which are independent for their natures of any thinker.

Frege's ontological Platonism is for the most part, as I have mentioned, the relaxed Platonism of a working mathematician. There are passages where Frege gives his ontological Platonism philosophical explanatory work. But for the most part he simply takes mathematics to have a subject matter. He thinks that it is obvious that this subject matter is independent of minds for its nature. (As I noted, his extension of his platonic view to senses and thought components generally is more problematic.) Even so there is a serious epistemic problem that can seem specially acute for ontological Platonism.

One of the most famous problems in philosophy is that of explaining how by merely thinking, for example in mathematics or logic, one can know something about a subject matter. The problem is usually motivated by noting that in empirical knowledge our thoughts are guided and controlled by causal relations to the subject matter. But in knowledge of abstract subject

matters in mathematics or logic, knowledge lacks this guidance and control. So how is it possible merely by thinking (however rationally) to know something about a subject matter? This problem was initially raised by Kant in his question "How is synthetic apriori cognition possible?".[47]

Kant produced an elaborate transcendental idealism to solve his own problem for mathematics and for purportedly apriori cognition in the natural sciences. Kant's theory is philosophically profound and a continuing source of philosophical insight. But its appeal to a putative capacity for pure intuition and its resort to the view that spatial and temporal structures are at bottom mind-dependent seems to me to disqualify it from serious candidacy for the truth.

The positivists responded to the problem by trying to protect empiricism. They held that mathematics and logic are only vacuously true and are not in any way made true by, or true of, a subject matter. They held that all genuine knowledge of a subject matter must be guided and justified by perceptual-causal relations to the subject matter. Kant anticipated this view by holding that logic but not to mathematics is vacuously true. This sort of response was, I think decisively, overthrown by Quine in mid-twentieth century.

Like the positivists, however, Quine tried to protect empiricism. Against the positivists, he held that truth in logic and mathematics is of a piece with any other truth–true of a subject matter and dependent for truth on a subject matter. He maintained, nevertheless, that logic and mathematics are ultimately known empirically through observations and empirical experiment. They thus remain under the control of causal relations to a subject matter. This account bears little relation to the actual practice of logic and pure mathematics, and fails to provide a plausible account of the epistemology of these subjects. Pure mathematics and logic rely on understanding and on proof for establishing their truths. They do not look to empirical observations for confirmation. There is no strong reason to think that their practices are not cognitively sound on

their own terms. In fact, they are paradigms of cognitively and scientifically powerful enterprises.

This brief catalog of responses suggests, correctly, that the Kantian problem has spawned cures that have been worse than the purported disease. All of these responses in effect give philosophy the role of criticizing, deflating, or limiting the applicability of the sciences of mathematics or logic. One of the sadder themes in the history of philosophy has been philosophy's tendency to invoke some less than obvious principle as ground for arrogating to itself the role of dictating acceptable practice, or otherwise radically re-conceiving the main results of sciences whose epistemic credentials are stronger than philosophy's. The epistemic credentials of logic and mathematics are certainly stronger than philosophy's.

Of course, no recipe limits philosophy's comment on science. Philosophy has sometimes produced critical insights. It has affected the development of science in positive ways. Still, successful global criticisms of scientific practice or successful wholesale re-conceptions of scientific results, especially since Newton, have been rare.

Frege's work suggests an approach to Kant's problem that is refreshingly free of the impulse to criticize or re-conceive mathematical or logical practice. His idea is that a condition on entering into the very practice of these subjects, indeed on having the very capacity of judgment, is bearing referential relations to a subject matter. Being an individual capable of judging, or having a mind capable of judging, requires connecting to a subject matter. It requires getting basic structural aspects of the subject matter right. Frege does not develop his views on this matter. In the discussion of logical objects in "Frege on Truth (1986) and in "Frege on Knowing the Third Realm" (1992), I try to elicit aspects of his thinking that suggest this direction, a direction which I believe to be profound and fruitful.

"Frege on Apriority" (2000) is motivated by my interest in understanding the history of

conceptions of apriority, as well as by a desire to understand certain peculiar aspects of Frege's own conception. The paper traces two conceptions of apriority in Leibniz. One of these conceptions is archaic, but interestingly connected to certain aspects of modern conceptions. The other is the basis for the modern conception. The paper also discusses Kant's refinement of Leibniz' conception. Kant's conception of experience as sense experience–a conception of experience seemingly more modern (or at any rate more specific) than Leibniz'–, makes his conception of apriori warrant or justification substantially the modern one: warrant whose force is independent of sense experience. Although Leibniz and Kant have very similar conceptions of apriority, their background philosophical views lead to very different accounts of how apriority is related to necessity and generality. In "Frege on Apriority", I bring out ways in which Frege's conception is closer to Leibniz'.

Unlike his two great predecessors, Frege is not very interested in necessity. He seems to gloss necessity in terms of apriority. Unlike the conceptions of apriority in Leibniz and Kant, Frege's conception of apriority does not explicitly center on independence of experience for justification. He explicates his notion of apriority in terms of a conception of generality: A truth is apriori if "its proof [its basic justification] can be derived exclusively from general laws, which themselves neither need nor admit of proof".[48]

Frege's explication of the notion is defective. It does not explicitly make reference to, or entail, anything about justificational independence of experience. He explicates it in terms of the generality of the principles on which a justification rests. This explication is, I think, vulnerable to counterexamples. What interests me, however, is the way it bears on Frege's views on geometry and arithmetic.

As regards geometry, Frege's apparent agreement with Kant on reasons why geometry is

synthetic apriori does not go very deep. Frege's conception of justification in geometry centers on the non-logical character of our capacities to "intuit" space. The justification is non-logical in that it concerns a special subject matter (space). Kant's conception of justification in geometry centers on the singular character of such intuition. Frege is committed to the justification's being ultimately general.

This difference affects their different views on knowledge (or cognition) of a subject matter. Kant takes synthetic cognition, cognition of a subject matter, to be necessarily grounded in singular thoughts that rest on (singular) intuition. Some intuition is non-empirical or pure, including intuition in geometry. But, for Kant, all substantive theoretical cognition rests for its warrant on singular cognition. Frege takes apriori cognition of a subject matter–hence cognition in logic and mathematics–to rest for its warrant on general cognition. Predication lies at the basis of apriori cognition for Frege. Cognition of abstract objects is warranted through the connection of objects to general predicative representations.

The relative merits of the two views as applied to different sorts of knowledge seems to me to be of great substantive interest. The relation between singular and general elements in representation, cognition, and warrant remains a matter of great importance in contemporary philosophy. It seems to me that both Kantian and Fregean conceptions of warrant and knowledge of subject matters--especially apriori knowledge of abstract subject matters--have significant contributions to make to contemporary thinking.

Frege is a resource for thinking about numerous central issues in contemporary philosophy. What I have found important and valuable in him differs from what his immediate successors found important, and even from what his best interpreters twenty-five to fifty years ago

valued. I believe that Frege's views on semantics, logic, sense-expression, thought, and knowledge are of immediate relevance to current philosophical thinking. I hope that these essays will contribute not only to a better understanding of Frege but also to creative work–constructive as well as historical--in our great subject.

Footnotes

1. The deflationist approach marks the work of Wittgenstein and his successors. Wittgenstein was one of Frege's earliest champions, as noted; but he was never strictly a positivist. So his motives for deflating philosophical problems are different–less centered on science. The same deflationary approach occurs in the work of Quine, who was the most prominent force in the overthrow of positivism, but who in this respect was an ally of Carnap and the positivists. Quine's work has played a large part in keeping alive the positivist demand for a theory of meaning and the positivist view that philosophical methods and problems that are not methods and problems within current science are to be doubted or dispensed with.

2. I believe that Russell's philosophy is also fundamentally driven by epistemic concerns. His approach to the theory of knowledge is closer to Frege's than to the positivists'. Russell was a rationalist and platonist about mathematics, like Frege. Like Frege he sees logic as a general science of "being". In my view, however, his basic doctrines about knowledge are extremely crude in comparison to Frege's. I think that Frege's conception of sense (as cognitive value) is on the right track for understanding cognition. I believe that Russell's account of denotation is of little value as the primary theoretical notion in understanding cognition. What I call Frege's pragmatic rationalism is vastly more sophisticated than Russell's theory of acquaintance.

3. The key papers were W.V. Quine, "Two Dogmas of Empiricism", in From a Logical Point of View (Harper and Row, New York, 1961; originally published 1953); and "Carnap and Logical Truth" (New York, Random House, 1966; originally published 1960) For discussion of them, see my "Philosophy of Language and Philosophy of Mind: 1950-1990", The Philosophical Review, vol. 101 (1992), pp. 3-51, and "Logic and Analyticity", Grazer Philosophische Studien, vol. 66 (2003).

4.He does call the relevant truths of logic "analytic" because he regarded such truths as "contained" within the basic logical laws. Traditionally containment was a type of analyticity. This idea embodies a mathematical error. It is a mathematical error because by Gödel's incompleteness theorems in higher-order logics such as second-order logic not all logical truths are contained in the basic logical truths in the sense that Frege intended. Frege counted such higher-order logics as part of logic. And he understood containment in terms of logical derivability.

5. Frege in effect gives an account of higher-order logic as well, but all the power essential to higher-order logic is, in effect, contained in second-order logic. Scholarship in the history of logic has, in the last few decades, emphasized numerous significant achievements in logic between Aristotle and Frege. But seen from the largest perspective, the picture that I have sketched still seems to me correct. The sketch relies on philosophical as well as historical oversimplification. It is a philosophical issue exactly what to count as logic. For some, Russellian type theory with an axiom of infinity counts as logic. For others, classical set theory counts as logic. On these conceptions, Frege's contributions to logic stop well short of completing logic proper (logic considered independently of its meta-theory). Frege's own logic included an axiom that turned

out to be contradictory, and its role in his theory has been filled by axioms in type theory or set theory. As I read the situation, however, the most nearly standard conceptions of logic take logical axioms not to be committed to an infinity of entities. These conceptions omit not only Frege's contradictory axiom but the axioms that were proposed as substitutes for it in type theory and set theory. So first-order and higher-order logics as they are now standardly conceived are less than what Frege proposed as logic. But they are all the work of Frege. Although Frege made serious contributions to the meta-theory of logic, much of what is now standard meta-theory (starting, for example, with the completeness theorem for first-order logic) was produced after Frege's career was completed.

6. Michael Dummett takes analytic philosophy to be defined by two doctrines: that a philosophical account of thought can be obtained through a philosophical account of language, and that a comprehensive account of thought can only obtained through a philosophical account of language, Origins of Analytical Philosophy (Cambridge, Massachusetts; Harvard University Press, 1994), p. 4. I believe that this characterization of analytic philosophy is interesting but much too narrow. It excludes Frege and Russell from being analytic philosophers, who would reject the second doctrine. There are many others in the analytic tradition who would follow them in this rejection. It fails to apply to yet others in the tradition who are silent on one or both of the doctrines. Dummett has taken views held by many analytic philosophers to characterize a tradition that is better characterized historically and by a loose sharing of methods and approaches. These do indeed center, to an unprecedented degree, on language. But attitudes toward language within the movement vary. Analytic philosophy is not a philosophical ideology.

7. Kant, Critique of Pure Reason, A65-9/B90-4.

8. The formulations occur in Foundations of Arithmetic, Introduction, p. x, sections 60, 62, 106. In some cases Frege states a necessary condition but uses it as a (qualified) sufficient condition.

9. Cf. "Frege on Apriority" and Postscript to "Frege on Truth". For substantially further discussion of these matters, see my "Logic and Analyticity", Grazer Philosophische Studien, vol. 66 (2003), pp. And Appendix to "Logic and Analyticity" contains a short history of the role of the intuitive notion of (formal) logical consequence in the history of logic.

10. Donald Davidson, Truth and Predication (Cambridge, Massachusetts; Harvard University Press, 2004). Davidson thinks that the key to unraveling the difficulty lies partly in denying that predicates are semantically related to anything, like properties, besides the objects that they are true of. I believe that this diagnosis is incorrect. The key to solving the problem lies at the logical/grammatical level, not at the ontological level. This is, in effect, the insight that Church completed, building on Frege. For more on this, see note 11.

11. Alonzo Church, "The Calculi of Lambda-Conversion" in The Collected Words of Alonzo Church, Burge, et. al. editors (Cambridge, Massachusetts; MIT Press, 2003; originally published 1941). Church himself took the result of attaching the lambda operator to an expression in order to produce a further expression with no free variables, as denoting a function. He took the

corresponding functional expression with free variables as denoting its values "ambiguously". I leave open whether this latter "take" is the best way to regard the situation. The important point, I believe, is that saturated expressions--expressions which are syntactically without open argument places or free variables, and which cannot, as they stand, function syntactically to make attributions or to take arguments–can denote functions or concepts. Church separates the syntactic role of being a predicate (or more generally, a functional expression) from the role of relating semantically to an entity that can be attributed (or more generally, to a function). I discuss philosophical grounds why Frege may have reasoned himself into his mistaken position in "Frege on Extensions of Concepts, from 1884 to 1903" (1984). For an illuminating analysis of Frege's mistake, which I largely accept–with qualifications deriving from the points made above–, see Terence Parsons, "Why Frege Should Not Have Said "The Concept Horse is not a Concept", History of Philosophy Quarterly, vol. 3 (1986), pp. 449-465.

12. Michael Dummett, Frege: Philosophy of Language (Cambridge, Massachusetts; Harvard University Press, 1981, second edition), pp. 7, 184, 196. Cf. also Dummett's paper, "Truth" (1959) in Truth and Other Enigmas (Cambridge, Massachusetts; Harvard University Press, 1979).

13. CITE end of "Frege on Apriority" for a discussion of this problem. I think that Frege does not fully solve it. His failure to solve it lies partly in the general failure of his logicism. But there are also intuitive, philosophical problems that I believe Frege never fully worked through.

14. Here and elsewhere I use "thought content" as the intuitive pre-theoretical notion that corresponds to Frege's theoretical notion of "thought" (in the sense of what is thought). Sometimes I use the term as an alternative to Frege's term, to indicate that I am writing about what is thought as opposed to the activity of thinking. When I use the term "thought content" on my own behalf, I do not invest it with a commitment to Platonism or with various other doctrines peculiar to Frege. The context will make this clear. I use the term "representational content" to apply to thought content, but also to other sorts of content, such as perceptual content, which need not be propositional. Again, I hope that the context will make the usage clear.

15. For more on the liar paradox, see my "Semantical Paradox" The Journal of Philosophy, vol. 76 (1979), pp. 169-198.

16. I believe that Frege provides some resources for solving it. I discuss these near the end of this section and in section III. I also indicate later that I think that the problem is arises differently for a subject matter known of and for a content of understanding and belief. I think that as applied to contents of understanding and belief, the problem is really a pseudo-problem.

17. Frege, "Logic", Posthumous Writings, Hermes, Kambartel, Kaulbach editors (Chicago, The University of Chicago Press, 1979), p. 145; cognate passage in the German: Nachgelassene Schriften (Hamburg, Felix Meiner Verlag, 1983), p. 157. Frege goes on, quite plausibly, to ignore the worry and claim that for the success of scientific enterprises, it is not necessary to solve this "mystery". He notes that it is enough that it be recognized that we do grasp thoughts and recognize them as true.

18. Michael Dummett makes much of how acute this problem is for Frege given his Platonism about thoughts, Origins of Analytic Philosophy, op. cit., p. 63-65, 107-108, 135-136. He claims, however, that since Frege takes thoughts to be expressed by language, the problem is not so acute since it resolves into how one understands a sentence. Much of what Dummett says about this latter problem is plausible. I believe, however, that Dummett is mistaken in thinking that the problem of understanding what it is to think can be solved only if thought is conceived in terms of understanding language. I believe that he is mistaken in assimilating Frege's notion of sense expression to his own conception of linguistic understanding. But primarily I think that he is mistaken in thinking that Frege's Platonism raises a problem about how understanding can traverse a great ontological divide. (Here I distinguish understanding a content from knowledge of a subject matter.) Frege can agree that understanding a thought content just is having certain linguistic or psychological capacities; they are what grasping a thought consists in. Any reasonable view must take the thought contents to be abstract. So all views face ontological questions about the relation between mental events in time and abstract representational kinds and types. But the questions are not those of how a thinker reaches out to thought contents. The ontological issues about abstract thought contents are quite different from issues about how we know mathematical objects. The difference lies in the distinction between expressing a sense and knowing something about a denotation. In the latter case, knowledge takes the objects as denotations or referents. In the case of understanding, the abstract entities are not objects of knowledge, nor is thought directed to them. Thought is type-identified in terms of them.

19. For a discussion of functions of sense, see my "Belief De Re" The Journal of Philosophy, vol. 74 (1977), pp. 338-362.

20. For a seriously wayward exposition of Frege's view on this last point, see Gareth Evans, The Varieties of Reference (Oxford, Clarendon Press, 1982), chapter 1. Evans imposes on Frege a view about object dependent reference in the senses of proper names that is clearly incompatible with quite a lot that Frege writes–including in "On Sense and Bedeutung".

21. Church called the relation of determination the relation concept of. Cf. Alonzo Church, "The Logic of Sense and Denotation" in The Collected Words of Alonzo Church, op. cit. (originally published 1951).

22. For eternality, see Frege, Posthumous Writings, op cit., p. 135. For thinker independence, see ibid, p. 134. Cognate passage in the German: Frege, Nachgelassene Schriften, op. cit., pp. 146-147.

23. Note the passage predates the Sinn-Bedeutung distinction. I think, nevertheless, that it is relevant to the point at issue.

24. Substantially the same remark about mankind having a common store of thoughts–for all the multiplicity of languages occurs in a footnote in Frege's essay "On Concept and Object"(1892).

25. Contrast Russell's remark: "It would be absolutely fatal if people meant the same things by their words. It would make all intercourse impossible, and language the most hopeless and useless thing imaginable, because the meaning you attach to your words must depend on the nature of the objects you are acquainted with, and since different people are acquainted with different objects, they would not be able to talk to each other unless they attached quite different meanings to their words." ("The Philosophy of Logical Atomism" (1918) in Bertrand Russell, Logic and Knowledge: Essays 1901-1950, Robert Charles Marsh ed. (Unwin Hyman, 1956).) Russell's view on this matter--along with his specially restricted epistemology and conception of acquaintance-- came to seem quaint, and certainly had little longer-term influence.

26. This conception of meaning is more specific than the very general sense in which sense, denotation, communal linguistic meaning, idiolectic meaning, intended meaning, and so on, are all conceptions of meaning in the broad sense that I discussed in saying, earlier in this Introduction, that Frege made contributions to our understanding of meaning and knowledge. I believe that Frege made contributions to our understanding of all of these conceptions of meaning, but his particular theoretical conceptions, sense and denotation, differ from the others.

27. Actually, several of the key points in "Sinning Against Frege" already appeared in section IV of "Belief De Re", op. cit..

28. Gottlob Frege, "Function and Concept" in Translations from the Philosophical Writings of Gottlob Frege (Oxford, Basil Blackwell, 1966), p. 29; "Funktion und Begriff" in Kleine Schriften, Ignacio Angelelli ed. (Hildesheim, Georg Olms Verlagsbuchhandlung, 1967), p. 132. The passage is on page 13 of the original article presented as lecture to Jenaische Gesellschaft für Medicin und Naturwissenscheft, Jena, Hermann Pohle, 1891.

29. Gottlob Frege, "On Sense and Bedeutung", the opening paragraph. Cf. Translations from the Philosophical Writigns of Gottlob Frege (Oxford, Basil Blackwell, 1966), pp. 56-57; "Über Sinn und Bedeutung" in Kleine Schriften, Ignacio Angelelli ed. (Hildesheim, Georg Olms Verlagsbuchhandlung, 1967), pp. 143-144; the passage is on page 26 of the original article, Zeitschrift für Philosophie und philosophische Kritik, 100 (1892).

30. There is a passage in "The Thought" (1918) where Frege seems to claim that some thoughts are not shareable--certain thoughts expressed through the first-person singular pronoun: in Collected Papers on Mathematics, Logic, and Philosophy Brian McGuinness ed. (Oxford, Basil Blackwell, 1984), p. 359-360; in "Der Gedanke", in Kleine Schriften, op. cit., p. 350; in the original article, in Beiträge zur Philosophie des deutschen Idealismus 1 (1918/19), p. 66. This passage is prima facie at odds with passages in which Frege seems to claim that it is of the essence of senses and thoughts (as opposed to ideas, Vorstellungen) that they not only not have bearers but that they are shareable. Cf. e.g. "On Sense and Denotation", Collected Papers, op. cit., p. 160; Translations of the Philosophical Writings, op. cit. p. 59; p. 29 in the original article, Zeitschrift für Philosophie und philosophische Kritik , op. cit.; or "Review: Husserl, Philosophy of Arithmetic" in Collected Papers, op. cit., p. 198; in Kleine Schriften, op. cit., p. 182; pp. 317-318 in the original review in Zeitschrift für Philosophie und philosophische Kritik 103 (1894).

The German in these passages is not completely unambiguous, and it is not fully clear that Frege is claiming that the essence of thoughts entails that they be shareable. Frege may be claiming only that many thoughts, or the thoughts at hand, can be shared. It seems to me that charitable interpretation suggests taking Frege not to have contradicted himself or changed his mind. It seems to me that Frege's claim about unshareability of certain particular senses is consistent with the rest of his philosophy. But his Platonism about such senses is extremely implausible for the sorts of reasons that I will discuss below. Dummett discusses the matter in a balanced way in "Indexicality and Oratio Obliqua" in The Interpretation of Frege's Philosophy (Cambridge, Massachusetts, Harvard University Press, 1981), pp. 120-124. He takes any claim of unshareability to be part of an "incoherent" doctrine. I disagree with this claim. In my view, Frege's remark that there are some unshareable thoughts is not of great importance in understanding his overall view. I believe, however, that it presents a further ground for not identifying sense with communal linguistic meaning.

31. It does not follow, as it is sometimes assumed, that Frege thought that there is some sort of reductive explanation of common languages in terms of idiolects. He does not discuss the matter. There are many possible positions to take on the relation between common languages and idiolects. The only thing that is clear is that Frege thought that some uses of language by individuals express senses that are not communally shared conventional linguistic meanings.

32. For an example of a fine interpreter struggling with the preconception about sense that I have been discussing, see Michael Dummett, "Indexicality and Oratio Obliqua", op. cit.. Dummett writes on the assumption that conventional linguistic meaning or significance is an "ingredient" of Frege's notion of sense (cf. pp. 100, 125-6, 128, 144). Dummett correctly sees that mode of presentation is an important part of Frege's conception of sense. But he sees Frege as running into tensions or difficulties because these two "ingredients" are in tension. He never provides convincing textual evidence that Frege's notion of sense is centrally concerned with conventional linguistic meaning. So his discussion of problems purportedly internal to Frege's doctrine seem to me to be consistently off base. On the other hand, his discussion is substantively subtle and, I think, largely right-headed about what is substantively right or wrong. Cf. also Dummett, Frege: Philosophy of Language, op. cit., p. 85ff.

33. Saul Kripke, Naming and Necessity (Cambridge, Massachusetts; Harvard University Press, 1972); Keith Donnellan, "Reference and Definite Descriptions", Philosophical Review 75 (1966), pp. 281-304; "Proper Names and Identifying Descriptions", Synthese 21 (1970), pp. 335-358 .

34. Cf. Frege, "Function and Concept", op. cit.

35. There are complexities in this case that I will not go into here. It seems to me plausible that any name must be associated by someone (if not the speaker, then someone the speaker depends on) with some descriptive or at least perceptual attributive element, even though the description or attributive element will almost never be complete enough in itself to determine a referent. The interplay between names or demonstrative devices, on one hand, and descriptive elements, on the other, is complex even in communal language use. It is even more complex in thought. For

discussion of some of these issues, see my "Five Theses on De Re States and Attitudes", forthcoming in a volume in honor of David Kaplan.

36. Frege's grounds for believing this principle are discussed in "Frege on Truth".

37. It is sometimes held that the name is obviously not ambiguous. It is held that the names "obviously" do not change denotations when they appear in such sentences: "Mark Twain" is still about Mark Twain regardless of what sentence one places the name in. I think that such criticisms are superficial and can be dismissed. Frege can certainly allow for an ordinary sense in which the name continues to be about Mark Twain in such contexts. But Frege's notion of denotation, like his notion of sense, is a technical or theoretical one. The notion of denotation is controlled initially by intuitions about reference, but also (and fundamentally) about intuitions about contribution to the truth value of sentences that contain them. One cannot dismiss Frege's view by appeal to such ordinary intuitions. One must evaluate Frege's theory in relation to its aims and the whole body of evidence that it is meant to explain.

38. Thus he is free to recognize that on some occasions, uses of relevant sentences do not attempt to specify fully the individual's thought, and may simply gesture at its general character in such a way as to satisfy practical communicative purposes on the occasion. Language use is quite complex and varied in this respect. On the other hand, I think that there is no plausibility to the view that language use never distinguishes between the truth values of sentences obtained by exchange of normally co-denoting expressions, or that such use never concerns the ways that individuals to whom attitudes are attributed are thinking. I believe that Frege's strategy is appropriate to aspects of many attributions of propositional attitudes in ordinary language. I believe that it is part of a correct theory for language that attempts to talk explicitly and fully about the propositional attitudes of individuals.

39. Gareth Evans, The Varieties of Reference, op. cit., pp. 90ff. holds that a memory caused in a normal way by only one of two objects that a person once saw, and not caused by the other, cannot refer to the object remembered if the person cannot distinguish the objects in some other way besides memory. I find no plausible defense of this counter-intuitive view, despite an elaborate theoretical structure meant to incorporate it. Evans' indefensibly restrictive views on singular reference in thought invade his account of perception as well as memory. I shall discuss these views elsewhere.

40. These matters are developed in detail in "Five Theses on De Re States and Attitudes", op. cit.

41. For development of these points, often with no special reference to Frege, see "Belief De Re", op. cit., and "Five Theses on De Re States and Attitudes" op. cit.

42. In the last three decades of the twentieth-century, beginning with the work on singular reference by Kripke and Donnellan mentioned earlier, work on natural kind terms by Kripke and Putnam, somewhat later work by me on anti-individualism, this assumption came to be rejected.

Cf. Saul Kripke, Naming and Necessity, op. cit.; Hilary Putnam, "Is Semantics Possible?" and "The Meaning of "Meaning""in his Philosophical Papers volume II (Cambridge, Cambridge University Press, 1975); and my "Individualism and the Mental", Midwest Studies in Philosophy vol 4 (1979), pp. 73-121 and "Other Bodies" in Thought and Object, Woodfield, editor (London, Oxford University Press, 1982). The rejection centered primarily on the meaning of expressions in empirical enterprises. Frege's work suggests an "anti-individualist" view that was mainly centered on considerations in mathematics and logic, although it has, I think, valuable applications to empirical cases. In "The Meaning of "Meaning"", Hilary Putnam mistakenly criticized Frege for holding the old view. This criticism seems to me as much dependent on confusions about the new view as about misconstruals of Frege, but it is in line with interpretations of Frege (e.g. by Dummett) as mainly concerned with conventional linguistic meaning. For substantive criticism of Putnam–since acknowledged as correct by Putnam–see "Other Bodies" in Thought and Object, Woodfield, editor (London, Oxford University Press, 1982).

43. Cf. my "Individualism and the Mental", Midwest Studies IV (1979), pp. 73-121; and "Other Bodies", in Thought and Object, Andrew Woodfield, editor, (London: Oxford University Press, 1982). Reprinted in The Twin Earth Chronicles, Pessin, ed., M.E. Sharpe, New York, 1996.

44. A skillful representation of this attitude may be found in Imre Lakatos, Proofs and Refutations (Cambridge, Cambridge University Press, 1976).

45. The idea that in the history of empirical science a term should be sometimes understood as expressing a constant concept or sense whose identity is not entirely fixed by current, often mistaken, scientific understanding has been illuminatingly emphasized in various essays by Hilary Putnam in Philosophical Papers vols. I-II (Cambridge, Cambridge University Press, 1975). The non-historical paper of mine that develops this idea most fully in connection with anti-individualism is "Intellectual Norms and Foundations of Mind" The Journal of Philosophy, vol. 83 (1986), pp. 697-720. This paper is closely associated in its substantive themes with "Frege on Sense and Linguistic Meaning". The notion of translational meaning in the "Intellectual Norms" paper is close to and was conceived as a version of Frege's notion of sense.

46. Of course, philosophy is another matter. I believe that there is apriori knowledge in philosophy. But it is much harder to base such a belief on a history of success than it is to base a belief in apriori knowledge in logic or mathematics. Still, I believe that the situation is less dismal than simple put-downs of philosophy commonly suggest--partly again, because relevant historical accounts are very complex.

47. It has been specialized to mathematics and elaborated by Paul Benacerraf in the late twentieth century. Cf. his "Mathematical Truth", The Journal of Philosophy, vol. 70 (1973), pp. 661-680. This matter is discussed in some detail in my "Logic and Analyticity", op. cit..

48. Gottlob Frege, The Foundations of Arithmetic, op. cit., section 3.