Postscript to "Frege and the Hierarchy" (2004)

Most philosophers who have thought about the matter take it to be unacceptable to be committed to a hierarchy of senses in accounting for embedded attributions of attitudes. Carnap rejected basic Fregean principles to avoid the hierarchy.[1] Dummett rejected what he claimed was an unimportant Fregean principle to avoid the hierarchy.[2] Early Terry Parsons maintained that Frege was not committed to the hierarchy, or at least could have avoided it without affecting the basic structure of his theory.[3] Davidson held that a language committed to a hierarchy is unlearnable.[4]

Church presented a detailed formalized language committed to the hierarchy. He believed, against Carnap, that substantial theoretical losses are incurred in avoiding it.[5] I believe that Church was right about this and that his point remains applicable to subsequent alternatives. I believe that the hierarchy is deeply grounded in Frege's standpoint, and that avoiding it requires giving up substantial parts of his theory. I also believe that avoiding it requires giving up principles that are motivated, powerful, and attractive—quite independently of Frege's maintaining them. Finally, I believe that it is not true that a language committed to an infinite hierarchy of senses is unlearnable.

I do not claim that there is no way around a sense hierarchy. I do believe that the reasons that favor postulating it have considerable plausibility. Senses are ways of thinking. So accounting for attribution of senses is accounting for attribution of (ultimately, thinking about) thought. I see Frege's account of

[1] Rudolf Carnap, *Meaning and Necessity* (1947), (Chicago: University of Chicago Press, 1967), 131.

[2] Michael Dummett, *Frege: Philosophy of Language*, 2nd edn. (Cambridge, Mass.: Harvard University Press, 1981), 267 ff.

[3] Terence Parsons, "Frege's Hierarchies of Indirect Sense and the Paradox of Analysis", *Midwest Studies in Philosophy*, 6 (1981), 37–57. I say "Early Terry Parsons" to distinguish this view from later developments, cited in n. 20 and 24.

[4] Donald Davidson, "Theories of Meaning and Learnable Languages", repr. in *Inquiries Into Truth and Interpretation* (1965) 2nd edn. (New York: Oxford University Press, 2001). Strictly speaking, Davidson claimed that Church's language of the "Logic of Sense and Denotation" is unlearnable. I think that this point is doubtful, even applied to Church's particular language. (The issue depends on how Church's subscripts are related to a prior understanding of an upwardly-functionally-determined hierarchy of canonical concepts.) I believe that Davidson and many others have taken the argument to apply to any language that invokes a hierarchy of sense. Dummett does, for example, in *Frege: Philosophy of Language*, p. 167. What is clear, I think, is that the version of the hierarchy I outline here is not vulnerable to Davidson's objection.

[5] Alonzo Church, "A Formulation of the Logic of Sense and Denotation" (1951) in T. Burge *et al.* (eds.) *The Collected Works of Alonzo Church* (Cambridge, Mass.: MIT Press, forthcoming).

attribution of thought as a contribution to a scientific attempt to account for thought, including actual propositional attitudes, as well as abstract thinkable *Gedanken*. I believe that at present there is no superior account of thought, or the attribution of thought—insofar as that attribution fixes on the nature of thoughts and attitudes. So I believe that the "sense" hierarchy should be taken seriously—not merely as a historical curiosity.

Frege's notion of sense was *not* pointed primarily toward understanding linguistic meaning as a common denominator in a socially shared language. It was pointed at understanding thought expressed by language. Such thought is sometimes shared among language-users in a community, sometimes not. Frege was mainly concerned that it be shared in a *scientific enterprise*.

I do not accept all Frege's views about sense. I think that the contents of thought are not in general eternal and mind-independent, as Frege believed. There are other errors in his view.

I think that Frege was in many respects, however, on the right track. He was right in thinking that words express thought components that are abstract and that are distinct from the ordinary denotations or referents of the words or thought components. Thought components are representational, or have aboutness properties, and constitute an epistemic perspective on those referents. I also believe that Frege's appeal to oblique contexts in attributions of propositional attitudes is part of a correct view.[6] Although I will write here of Fregean senses, the structural points that I make about them are, in the main, applicable to abstract representational contents of thought. Most components of representational thought contents are what I call concepts.[7] Some

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[6] In this paper, except where context makes another usage clear. I will use "oblique context" in such a way as not to pre-judge the issue between Method I and Method II formalizations of natural language. Method I formalizations take the same expression to express different senses and have different denotations in different contexts. Method II formalizations treat the same word-forms in natural-language identity and that-clause contexts as ambiguous, in such a way as to require formalization by different expressions. An oblique context O in this sense is a position in a natural-language sentence (construed in a certain way) in which intuitively substituting different word-forms that have the same denotation in certain other contexts (such as the contexts of singular terms in identity statements) is not guaranteed to preserve the truth-value of the whole sentence in which context O occurs. Thus, intuitively, substitution of "Samuel Clemens" for the second occurrence of "Mark Twain" in the sentence "Al believes that Mark Twain is identical with Mark Twain" is not guaranteed to preserve the truth of the whole sentence (on a certain construal of the sentence), although the two names have the same denotation in the identity sentence "Mark Twain is identical with Samuel Clemens". It is fundamental to Frege's method—and a view that I regard as fundamentally on the right track—that in oblique contexts a word-form denotes a sense, not the denotation that the same word-form has in contexts like those in an ordinary identity statement. I use "customary sense" similarly. It applies to the sense expressed by word-forms as they occur in non-oblique contexts. "First indirect sense" applies to the sense of an expression as it occurs in an unembedded oblique context. This usage leaves it open whether customary senses are identical with first indirect senses (and whether first indirect senses are identical with second indirect senses, and so on). According to the hierarchical view, all of these are distinct.

[7] This usage is, of course, distinct from Frege's use of "concept" (*Begriff*). My usage is closer to that of Alonzo Church, "A Formulation of the Logic of Sense and Denotation".

are what I call applications. Representational thought contents are true or false. Substituting "representational thought content" (used to include components as well as whole thought contents) for "sense" sometimes skirts common misunderstandings of Frege's view.

I will not discuss all these issues. I do want to say a little more than I did in "Frege and the Hierarchy" about how to think about the hierarchy in an intuitive manner. I also want to explain why a language committed to it is learnable. Finally, I want to discuss some more recent objections to the hierarchy and a response to my paper that attempts to avoid the hierarchy.

I will assume Frege's view that in ordinary, non-embedded attributions of propositional attitudes, expressions within natural-language that-clauses, occurring in positions where free substitution of co-denotational expressions is problematic, denote their customary senses, not their customary denotations. Consider again

(1) Bela believes that Opus 132 is a masterpiece.

Let us assume with Frege that the whole that-clause denotes the customary sense (thought content) of the words "Opus 132 is a masterpiece". The words within the that-clause denote their respective customary senses. These senses are components of the customary sense denoted by the that-clause.

It is evident that in using (1), one's understanding of the *customary* senses of the words in the that-clause plays a role in picking out those very customary senses. On Fregean principles, the customary sense cannot determine itself, since the customary sense determines a truth-value; and a given sense determines a unique s-denotation.[8]

Frege allows words in natural language to shift their senses and denotations with linguistic context. But he takes senses themselves to determine a unique entity (a unique s-denotation), if they determine any entity. He explicitly denies that the *sense* of an expression within the that-clause—its indirect sense—is the same as its customary sense.[9]

It seems to me that inasmuch as the entities denoted by words in the that-clause are different from those denoted by the same words in customary, direct contexts, it is intuitively evident that the modes of presentation associated with words in that-clauses are different from those associated with those words in direct contexts. This seems to me intuitively evident independently of appeal to Frege's principles. Thus "Opus 132" as it occurs in

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[8] S-denotation or determination is the relation between a sense and the entity that it uniquely determines. That entity is denoted by an expression that expresses the sense. The relation of s-denotation or determination is the relation that Church called the *concept-of* relation. Determination of an s-denotation or a *determinatum* is the non-linguistic analog (between senses and entities rather than symbols and entities) of denotation. For Frege, and I think sometimes in fact, it is more basic than the relation that a linguistic expression bears to its denotation by way of its sense. Sometimes thought is more basic than language. Sometimes what is in fact a sense just happens to be expressed by a word. The thought content itself sometimes antedates expression through language.

[9] Gottlob Frege, "On Sense and Denotation", O 37.

customary contexts specifically denotes a string quartet. In a that-clause (occurring obliquely) it specifically denotes a way of thinking about, that string quartet. The sense, or way in which these different denoted entities are thought about should also differ. Similarly, "is a masterpiece" denotes a property of works of art, or (if you insist) a set of works of art, in customary contexts. In that-clauses, it denotes a way of thinking about that property. Ways of thinking about the property—modes by which thought is presented with the property—and ways of thinking about a way of thinking about the property are surely different.

One should stop thinking of senses as conventional meanings of words and sentences. Sometimes they are; sometimes they are not. Senses are ways of thinking, perspectives on entities presented, or purportedly presented, to the mind. The perspectives or ways of thinking about the denotations, or *determinata*, are different in the two cases. Given that these ways of thinking are not indexical ways of thinking, and given that the *determinata* are in the actual world, the difference in denotations or *determinata* argues very strongly for their being presented to the mind by different ways of thinking about them. To talk of a sense here as having multiple determinations or s-denotations is, I think, to lose sight of the fundamental role of sense in type-identifying specific cognitive perspectives on a subject matter.

I believe that "Frege and the Hierarchy" shows that if certain Fregean principles governing sense and denotation are maintained, then either in the object-language or in the metalanguage that explains the truth conditions of the object-language, one must distinguish the indirect sense of an expression (the sense it expresses in an unembedded that-clause) from its customary sense. I shall return to the details.

There does remain the fact that in using a that-clause one relies on one's understanding of the *customary* senses of the words within it in determining the that-clause's indirect denotation. Here the senses of the relevant words determine the representational content of Bela's thought. To understand the indirect sense of "Opus 132", one mainly has the customary sense (one's customary way of thinking about that string quartet) to go on. So the senses of the words as they occur in the unembedded that-clause must bear a relatively simple relation to the senses that they express when they occur in an embedded that-clause—the indirect senses.

One can see something similar going on in the way quotation marks are normally used. There is a legitimate but non-ordinary usage of quotation marks, especially common among philosophers of logic. On this logician's usage, quotations yield a name of the word-shape that they enclose, and one's understanding of the quotation abstracts completely from any meaning that the quoted word-shape might have. Of course, there are also uses of quotations of meaningless or foreign words that are not understood. But normally when quotation marks are used, there is an associated assumption that the quoted expressions have a meaning or sense in the language in which

the quotation marks are used. This is why quotations are ordinarily translated.[10] For the sake of argument and illustration, let us include this assumption about the meaning or sense of the quoted word in what we shall call the normal construal or understanding of the quotation.

Consider this construal of the quotation expression " 'Opus 132 is a masterpiece' ". The expression denotes the expression "Opus 132 is a masterpiece"; and this expression is understood as having its usual meaning in English. The quotation expression " 'Opus 132 is a masterpiece' " does not have the same normal construal as the quoted expression "Opus 132 is a masterpiece". But one's understanding of the quotation expression depends only on understanding the shape and meaning of the quoted word, and on mastery of quotation marks. One's understanding of the quotation expression is determined by the sense of quotation marks, how many iterations of quotation marks are involved, and the sense and form of the words quoted.

That-clauses—at least insofar as they are used to characterize the *nature* of the propositional attitude—work like the normal construal of ordinary quotation expressions, just discussed, except that they are used to denote only the sense, or representational content, of words within them. Unlike quotation names, that-clause names do not denote the words *per se* at all.[11] Unlike quotation names, that-clauses enable one to denote just the sense shared by different words—for example, words in different languages. A non-embedded that-clause expresses a new sense that determines the customary sense. This new sense is the indirect sense of the word. This sense determines the customary sense of the words within the that-clause, whereas the sense of the words within the that-clause determine the customary denotations of the words. Normally these customary denotations are not senses at all. In fact, the logical type of a sentence standing alone differs from the logical type of the that-clause name. In Frege's view, sentences have truth-values as denotations. But, arguably, sentences do not denote anything at all: they simply have truth-values. An analogous point holds of some expressions within the sentence and the expressions that denote their customary senses: Their denotations may be of different type than the senses that they express. This

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[10] For discussion of some of the complexities associated with this point, see my "Self-Reference and Translation," in M. Guenthner-Reutter and F. Guenthner (eds.), *Meaning and Translation* (London: Duckworth, 1978), 137–153.

[11] This view represents a change of position, on my part, from the position in "Self-Reference and Translation". I changed views relatively soon after writing that paper. At the time I wrote that paper, I was attempting to work out a Carnap-like position according to which words occurring obliquely in that-clauses denote words construed as having definite meanings. I have subsequently given up that view in favor of the view maintained by Frege and Church—that such words normally denote abstracta that are not necessarily linguistic entities, in fact, normally *not* linguistic entities. I do continue to believe that the rebuttal to Church's translation argument that I gave in "Self-Reference and Translation" both prevents the argument from being decisive, and indicates something important about the nature of quotation. The reason for maintaining the Church–Frege view about the denotation of words that occur obliquely is not Church's translation argument. It is that the Fregean view is more natural and provides a deeper account of what is really at issue in attributions of propositional attitudes—at least when those attributions are concerned with the natures of the attitudes being attributed.

is further reason to take the indirect senses to be distinct from the customary senses.

There is a function from the indirect sense of each expression to the customary sense of that expression: Sense determines denotation (or rather s-denotation, or *determinatum*), as usual. That is old news. The important thing to notice is this: The sense of the that-clause (understood to contain only oblique occurrences)—the first indirect sense of the sentence within the that-clause—is dependent on nothing other than the customary sense of the words within the that-clause, together with the sense of the that-clause-forming expression (expressed in English by "that") that is applied to those words.

We form a canonical name of the customary sense by applying the that-clause-forming expression ("that") to the sentence that follows. The word "that" is not crucial. The crucial things are the position of the expression in the subordinate clause and the intuitive point of the position to help specify a perspective or way of thinking. The indirect sense is uniquely fixed by this way of denoting the customary sense, assuming that the words in the that-clause occur obliquely. The sense of the canonical name can be uniquely recovered from the customary sense. There is a function from the that-clause name's denotation—the customary sense—to what I shall call the *canonical sense* that determines that customary sense. We understand the indirect sense, and "know what it is" in the sense of comprehending it, if we can both use the that-clause-forming expression ("that") and understand the sense of the sentential expression to which it applies. Nothing more is needed. I shall elaborate this point shortly, in the context of a formalization of the sense hierarchy.

So far we have arrived at two levels of senses—the customary sense and the indirect sense. Many philosophers have thought that one should stop with one level of sense, or at most these two. My paper shows, I think, that one cannot do so without giving up basic Fregean principles.

Given the way in which senses of these canonical names work, it seems to me evident that there is no particular problem about *learning* an infinite hierarchy of canonical senses expressed by such canonical names. The sense of a canonical name that denotes a sense at level n ($n \geq 1$) is a sense at level $n + 1$.

There is a *function* from a sense at level n to the *canonical* sense at level $n + 1$ that determines, or s-denotes, the sense at level n. Let us call this function "*the canonical sense* function".[12] Let us introduce the one-place

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[12] This requirement can be modified. There may be reasons to allow for a more fine-grained conception of canonical sense and canonical name. Thus it has been claimed that word-forms with the same customary sense are substitutable in unembedded oblique contexts, but are not substitutable in embedded oblique contexts. Cf. Benson Mates, "Synonymity", in L. Linsky (ed.), *Semantics and the Philosophy of Language* (Urbana, Ill.: University of Illinois Press, 1952). To allow for such a view, one needs to allow that the indirect senses of word-forms with the same customary sense can differ; and word-forms that denote the same customary senses in oblique contexts can denote different

functional expression "C". The function denoted by "C" takes senses as arguments and yields, as values, *canonical senses* that determine or s-denote the argument sense. When "C" is syntactically applied to a *canonical name* of a sense s, it yields a canonical name of the canonical sense that determines that sense s.[13]

This functional expression could be roughly glossed "the canonical sense that determines", *except* that "C(s)" must be understood not as a description but as a canonical name.[14] "C" can, of course, be iterated.

Before discussing the sense of "C", let me say a bit more about the syntax of the language in which I conceive "C" as occurring. "C" will be used in formalizations of embedded oblique contexts in natural language. The that-clause-forming expression in English, "that" (like quotation marks), is not clearly a functional expression.[15] Nevertheless, the that-clause-forming expression "that" has many of the characteristics of a functional expression, especially in occurrences that embed other that-clauses. In a Method II formal language, one needs names for the customary senses of ordinary first-level expressions. Then a functional expression like "C" can syntactically apply to these names, and iterate.

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indirect senses in singly embedded oblique contexts. I believe that this view can be accommodated by allowing that the indirect sense of a word-form is a function of the customary sense together with the word-form itself, or whatever is common to exact translations of the word which are finer-grained than sameness of customary sense. Then we would have a conception of a fine-grained canonical sense whereby understanding the fine-grained canonical sense that determines a customary sense requires understanding not only the customary sense, but what word expresses it. The conception of a fine-grained canonical name would parallel this notion of fine-grained canonical sense. Most of the examples used by Mates and his critics (including Church) seem to me to depend on conflating sense and conventional linguistic meaning. Thus *fortnight* and *period of fourteen days*, or *physician* and *doctor*, have respectively the same conventional linguistic meaning. But I think that they are commonly used to express different modes of presentation, different ways of thinking—hence different senses. Cf. my "Belief and Synonymy", *The Journal of Philosophy*, 75 (1978), 119–138. These matters need better sorting out. Nevertheless, it seems to me possible that there will be an explanatory use for the Mates-like position. I wish here just to indicate that the sense hierarchy can be conceived in such a way as to accommodate that position.

[13] I am comfortable in calling complex expressions formed in functional ways "names". They are not descriptions. They are rigid. I think that they are similar to numerals in the base-ten system larger than "9". But if one wants to require that names cannot have functional structure, or if these canonical expressions fail some other linguistic test for names, I am willing to call them "canonical designators". The key facts about them are that they are non-descriptive, rigid expressions, whose denotation can be computationally determined. There is a further feature of canonical names of senses that I am about to articulate in the *Principle for Canonical Names of Senses*. This feature or principle distinguishes them from other canonical names or designators, including numerals. My thinking about this matter goes back to "Self-Reference and Translation". Cf. note 10.

[14] Actually, the situation is slightly more complex. If "C" syntactically applies to any singular term T that denotes a sense, the resulting syntactical unit is a term that denotes the canonical sense. This is the canonical sense that determines the sense denoted by the singular term T. If "C" is syntactically applied to a canonical name of a sense, the resulting syntactical unit is a canonical name. If "C" is syntactically applied to any singular term other than a canonical name of a sense, the resulting syntactical unit is not a canonical name; and the sense of the whole functional expression is not itself a canonical sense.

[15] For quotation marks to be a functional expression, the expressions occurring inside quotation marks would have to be seen as naming, or otherwise denoting, themselves.

Suppose that the primitive expressions of the formal language initially do not include expressions that denote the senses of any of the language's expressions. Suppose that we add simple canonical names for the customary senses of each of the finitely many primitive expressions that are not themselves canonical names of senses. So we have doubled the number of primitive expressions. Then we add the functional expression "C". Thus we have canonical names for the customary senses of the primitive expressions, canonical names (via an initial functional application of "C") for the canonical senses that determine the customary senses, further canonical names (via one iteration of "C") for canonical senses that determine the canonical senses that determine the customary senses; and so on.

Later I will introduce a device for forming canonical names for the customary senses of syntactically complex expressions. Thus there will be means of composing canonical names for the customary senses of primitive expressions into canonical names for the customary senses of complex expressions. Further, there will be compositional principles for "C" that enable one to form canonical names for complex senses out of canonical names for the components of the complex. Thus, there will be principles for composing canonical names of complex senses to produce canonical names of the senses at any finite level of sense, from the first level, the level of customary sense, onward.

I turn now to the sense of "C". I maintain the following

Principle for Canonical Names of Senses: *The canonical name of a sense can be understood only if the sense that it names is understood.*

For example, to understand the sense of the canonical name for the customary sense of "3", one must understand the customary sense of "3".

I believe that the relevant canonical names obey a

Stronger Principle for Canonical Names of Senses: *To think the sense of a canonical name of a sense, one must simultaneously think the lowest-level (ultimately, customary) sense in the downward hierarchy associated with the canonical name.*

For example, to think the sense of the canonical name "C(C(s))", where "s" is a canonical name of a customary sense, one must think the customary sense denoted by "s".

These principles should be understood in light of the earlier discussion of quotation marks and of that-clauses. I regard expressions occurring obliquely in that-clauses as canonical names of senses. In unembedded that-clauses, such expressions are canonical names of customary senses. Customary senses are ways of thinking that determine or s-denote ordinary objects and properties (like string quartets, or violins, and properties of them). The customary sense or way of thinking that is denoted by an obliquely occurring expression in a that-clause is presented in a canonical way. The sense is thought about

from the perspective expressed by a first-level canonical name of it. These canonical names have different senses from senses of other expressions that denote the same customary senses.

It helps here, as elsewhere, to remember that senses are fundamentally cognitive perspectives or cognitive modes of presentation. The canonical sense or way of thinking expressed by the first-level canonical name can be understood only by *simultaneously understanding* the sense or way of thinking that that expressed sense, or way of thinking, determines. (It determines the customary sense that the first-level canonical name denotes.) Thus to understand the sense of the first-level canonical name "Opus 132", as it occurs obliquely in an unembedded that-clause, one must simultaneously understand the denoted customary sense of "Opus 132". Similarly, to understand the sense of the first-level canonical name that *formalizes* "Opus 132", as "Opus 132" occurs obliquely in an unembedded that-clause, one must simultaneously understand the customary sense that the formalizing canonical name denotes.

The *Principle for Canonical Names of Senses* maintains in effect that understanding the canonical sense of any canonical name in the hierarchy obtained by iteration of "C", or of that-clauses, partly consists in understanding the customary sense of the expression at the bottom of the hierarchy. Here the analogy to the normal construal of quotation marks seems especially apt. To assume an understanding of the quoted expression, one must understand the expression at the bottom of the hierarchy of iterated quotations. Thus to translate """"masterpiece"""" into another language, one must understand "masterpiece".

It is clear that in attributing a way of thinking to someone in the that-clause fashion, one must understand that way of thinking. That is, in making reference to a way of thinking in an unembedded that-clause-type attribution, one must have a capacity to think with the way of thinking (the customary sense) that one attributes. One makes essential use of one's first-level mastery. But it is also clear that a capacity to attribute a way of thinking goes beyond the capacity merely to think with that way of thinking. It is one thing to be able to think a thought and another to be able to attribute the thought to someone else, or to oneself. There are stages of development when a child can do the first and not the second. Indirect senses expressed by canonical names of customary senses are ways of thinking or modes of presentation that mark or type-identify this additional attributive capacity that is parasitic on the capacity to engage in the root, first-level way of thinking. Higher-level ways of thinking expressed by higher-level canonical names make use of the same root, first-level capacity. They mark the additional difference, for example, between attributing an attribution and attributing a first-level way of thinking. It is one thing to attribute a thought and another to be able to think about someone else's, or one's own, attribution of a thought. Here again, different modes of presentation mark the different intellectual capacities. General

theoretical principles, which I will discuss later, motivate distinguishing indefinitely many levels of potential perspectives, or attributional ways of thinking above these three levels. All such ways of thinking are canonical in that they are fixed (a) by what it is to understand and employ any attribution *at a given level*, and (b) by what it is to understand and employ the particular first-level ways of thinking that underlie the attributions.

There are some elementary things to re-emphasize about the sense hierarchy so far postulated. As with any sense of an expression, there is a downward function from a canonical sense to the denotation that it determines. Canonical senses determine unique senses. The canonical sense of any canonical name determines or s-denotes the denotation of the canonical name.

What is special to *canonical senses* is that there is an upward function from senses to *canonical* senses that determine them. Thus there is a "backward road" from senses to their canonical senses. The *Principle for Canonical Names of Senses* (and its stronger counterpart) should be helpful in getting an initial grip on this idea. Canonical senses are ways of thinking about senses that are grounded in a grasp of the senses that are thought about. I will be elaborating plausible principles that bring out that thinking about senses in certain contexts requires that the perspective on a sense—the way of thinking about it—be distinguished from the sense being thought about. This remains so even though the perspective is (largely) fixed by the subject matter, the sense, or way of thinking, thought about.

We began by introducing "C" as denoting a function from senses to canonical senses that determine them. Even granting that for each sense there is a unique canonical sense that determines it, there are many ways of thinking about this function. So this introduction does not in itself fix or explain the sense of "C". One might wonder which senses the canonical senses are. Or one might feel that although one understands the customary sense, it is mystifying what further canonical sense determines this customary sense.

I have gone beyond the introduction of "C" in terms of a description of the function that it denotes. I have indicated precisely what materials are used in understanding the higher-level senses. First, to understand the sense of a canonical name, one must know how to use names and have an understanding that distinguishes the sense of a canonical name from the sense of a description—even a description like "the sense of 'Opus 132' ". Second, I have invoked the *Principle for Canonical Names of Senses*. To understand the sense of a canonical name, one must also understand the customary sense at the bottom of the hierarchy within which the name is situated. Thus to understand the canonical sense that determines a customary sense, one must understand the customary sense. There is strong ground to distinguish the canonical sense that determines a customary sense from the customary sense itself. The customary sense is a way of thinking specifically about such things as string quartets or the property of being a masterpiece. The canonical sense that determines the customary sense is a way of thinking specifically

about ways of thinking about string quartets or being a masterpiece. These ways of thinking–the ones about the quartets and such, and the ones about ways of thinking about quartets and such–are clearly different.

Now we go further in explicating the sense of the canonical names. To understand the initial layer of canonical names of the customary senses of primitive expressions in the language, one need only understand unembedded oblique occurrences of the expressions in natural language that these canonical names formalize. In understanding those natural-language expressions, one uses those expressions as canonical names (not descriptions). And one understands them by understanding the customary senses of those expressions (which are denoted by the expressions), while simultaneously understanding those expressions as denoting ways of thinking, not the entities (like string quartets) determined by the customary senses. Thus one is not using those customary senses as ways of thinking. One is using them as aspects of ways of understanding the ways of thinking that determine those customary senses. The full understanding of ways of thinking that determine those customary senses depends on comprehending attributions expressed (whether contingently or essentially) in unembedded oblique occurrences in natural language. Such understanding constitutes the additional layer of intellectual capacity discussed earlier. I believe that the senses of the initial layer of canonical names cannot be explained in any further way. No further way is needed.

To understand higher-level senses in the hierarchy, one must understand iterations of oblique contents signaled in English by iterations of that-clauses. Understanding "C" is essentially understanding the formation of a canonical name that meets the conditions of the preceding paragraph but also adds a level of perspective on the preceding two levels of senses. It is the understanding that accrues from understanding an iteration or embedding of a that-clause—from having the intellectual sophistication to take up the perspective of an additional layer of attribution. The key element is not the "that" itself. What matters is an understanding of canonical names informed by the *Principle for Canonical Names of Senses* and of the level of embedding of perspectives on perspective. "C" marks an embedding or a raising of canonical naming perspective. Iterations mark further levels of embedding. We understand "C" insofar as we understand the iterations of oblique occurrences in that-clauses that it helps formalize.

Thus I believe that it is necessary and sufficient in understanding canonical names formed with "C" that one understand (a) the customary sense, (b) a canonical naming perspective on that customary sense—a perspective of the sort involved in understanding sense-naming expressions occurring obliquely in unembedded natural-language that-clauses—and (c) a capacity to keep track of levels of iteration or embedding. Mastering the hierarchy of canonical names requires and involves nothing more. I believe that any other specification of the sense of "C", or of canonical names formed with it, is likely to be inaccurate or misleading. "C" mimics the intuitive understanding

of natural-language embeddings of oblique contexts. If one understands that-clauses and their iteration, one understands the sense of such a functional expression, like "C", for building canonical sense-names.

What "C" brings out is that understanding higher-level senses is functionally dependent on (hence only on) four elements: (1) understanding the customary senses of the primitive non-sense-naming expressions; (2) understanding a finite number of canonical names of those customary senses (which requires only understanding a canonical name that "gives" the denoted sense by co-occurring with an understanding of the denoted sense);[16] (3) understanding (and keeping track) of levels of embeddings;[17] and (4) understanding principles for functionally composing canonical names of the senses of complex expressions from canonical names of the senses of simple expressions.

Thus it is necessary and sufficient to understand "C(C(<Opus 132>))"—where "<Opus 132>" is the canonical name of the customary sense of "Opus 132"—that one understand the larger expression and all its component singular expressions as canonical names, understand the underlying customary sense, and understand double embedding of an occurrence of "Opus 132".

Clearly, one can master this structure with finite resources: a finite number of canonical names for customary senses, functional applications of "C" to

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[16] Understanding a canonical name is to be distinguished from understanding a definite description. Unlike ordinary definite descriptions, canonical names are rigid designators. Unlike even rigidified descriptions like the "actual customary sense of "Opus 132"", they do not make reference to any general properties like "sense" or any entities other than senses (like words). These particular canonical names are special in two ways. One is that they involve situating the named entities in a structure, in something like the way that understanding the numerals involves situating the numbers in a structure. As with numerals, the canonical names that contain "C" are structurally complex, and the complexity matches aspects of the structure of entities that are named (by way of understanding the named sense). The other way that these canonical names are special is that they "give" the sense: understanding them requires understanding the sense that they name, and ultimately the root sense, in any given hierarchy. The key matter here is the *Principle for Canonical Names of Senses*. Cf. note 13.

[17] Thus "C" just raises the level of perspective or embedding. "C" and its iterations are analogous to marking the number of digits in a base-ten numeral. The level of perspective has semantical import in itself, as does the number of digits. The difference is that the numerals do not relate to their denotations in as initimate ways as canonical names relate to *their* denotations (cf. the *Principle for Canonical Names of Senses*).

Here is a way that the level of embedding can matter semantically. In the sentence

(IS) Igor believes that Arnold's favorite proposition is something such that Bela believes it,

one can take "Arnold's favorite proposition" to occur obliquely. We reporters cannot substitute just any co-denoting expression for it in specifying Igor's beliefs. But from Igor's point of view, the term (or its sense counterpart) could occur transparently in Igor's thinking about Bela's beliefs. Arnold may not in fact have a favorite proposition, but Igor thinks he does. And Igor will allow substitution of any expression that he regards as co-denoting with "Arnold's favorite proposition" (or any sense that he regards as co-determining with the sense of "Arnold's favorite proposition") in his specification of Bela's belief. Thus our perspective must be distinguished from Igor's, and Igor's, in turn, from Bela's, even though the two perspectives which the expression "Arnold's favorite proposition" plays a role in specifying (Igor's and ours) understand the relevant canonical name of their respective senses fundamentally in terms of the root sense—the customary sense of "Arnold's favorite proposition".

those initial canonical names, and recursive principles for combining these canonical names into canonical names for the senses of syntactically complex expressions (like the senses of sentences).

There is no more difficulty in learning a language committed to an infinite hierarchy of senses than there is in learning a language that iterates quotation marks. There is no more difficulty in learning a language committed to an infinite hierarchy of senses than there is in learning a language involving base-ten canonical names for the numbers. The higher levels of canonical senses are determined by the customary sense together with iterations of the that-clause construction (or "C"). So all the talk by Davidson, Dummett, and others about our not knowing what the indirect senses and higher-level senses of expressions are, or of being unable to learn the hierarchy, is wayward and unsupportable.

According to Frege, each level of iteration of a that-clause (assuming that we are dealing with oblique contexts within that-clauses) yields a higher-level canonical name and expresses a higher-level canonical sense.[18] Thus

(2) Igor believes that Bela believes that Opus 132 is a masterpiece

attributes to Igor belief in the thought content that Bela believes that Opus 132 is a masterpiece. The that-clause that includes (1), as it occurs in (2), denotes the customary sense of (1). The customary sense of the expression (1) is composed of the customary sense of "Bela", the customary sense of "believes", and the customary sense of the that-clause "that Opus 132 is a masterpiece". The customary sense of this that-clause is a canonical sense that determines the customary sense of the sentence "Opus 132 is a masterpiece". So in (2) the that-clause "that Opus 132 is a masterpiece" denotes this level-2 canonical sense or cognitive perspective. It expresses a level-3 canonical sense that determines this level-2 canonical sense. And so on.

The higher-level senses denote ways of thinking about the lower-level senses. At each level one takes up a new attributional perspective on the level below. These ways of thinking, at different levels of canonical sense, differ, however, entirely in the level of embedding of attribution. Thus the differences are needed by, but are exhausted by, the levels of logical attribution. The fundamental "line" of content for each canonical upward route from a customary sense is fixed, *except for the levels of attribution*, by the customary sense itself.

Let me speak to two intuitive objections to the hierarchy. I believe that these objections are easily met, given the upward and downward functional relations among canonical senses.[19]

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[18] Gottlob Frege, *Philosophical and Mathematical Correspondence*, ed. G. Gabriel *et al*. (Chicago: University of Chicago Press, 1980), 153–154; *Wissenschaftlicher Briefwechsel*, ed. G. Gabriel, F. Hermes, F. Kambartel, G. Thiel, G. Veraart (Hamburg: Felix Meiner, 1976), 234–237; Frege to Russell, 12/28/1902.

[19] Both of the objections that follow are given by Christopher Peacocke, "Entitlement, Self-Knowledge, and Conceptual Redeployment", *Proceedings of the Aristotelian Society*, 96 (1996),

Suppose with Frege that in (2) we make reference to a certain mode of presentation, partly fixed by a level of logical attribution of the belief content which Igor ascribes to Bela. Thus our attribution attributes to Igor a thought that contains a canonical sense that determines the customary sense of (1). The customary sense of (1) contains a canonical sense that determines the customary sense of "Opus 132 is a masterpiece". Assume that (2) is true. Thus Igor's belief contains a canonical sense that determines the (presumed) content of Bela's belief. And our attribution to Igor contains a (third-level) canonical sense that determines the second-level canonical sense that Igor (presumably) uses to think about Bela's (presumed) thought content, customarily expressed by "Opus 132 is a masterpiece".

One might object that this is an unintuitive description of the attribution in (2). One might object on the ground that for (2) to be true, (a) Igor must be thinking that one of Bela's beliefs has the content *Opus 132 is a masterpiece* and (b) in thinking this, Igor is employing the same content in thought as Bela would if he were to think that Opus 132 is a masterpiece. It might be further held (c) that Igor is not thinking of the content in some indirect way as the first content asserted on such and such a page of a particular, named book.

This objection is ineffectual. Frege would agree with (a). That is in effect just what (2) says. That is common ground. Frege would agree with (c). Canonical names do not denote "indirectly" as definite descriptions do. I believe that I have already developed this point in sufficient detail. Frege would agree with (b) *in a sense*. Igor could not attribute to Bela a belief through a canonical name of that belief content—or canonical sense that determines the belief content—unless he was also thinking the content. To attribute to Bela the content that he does, he must in doing so also think the content of Bela's presumed belief—the customary sense of "Opus 132 is a masterpiece". That is how canonical names of content work, as I have been explaining.

From a canonical perspective, we attribute to Igor a canonical perspective on Bela's presumed belief content. The perspective that we attribute to Igor determines that content. Our perspective determines the perspective that we attribute to Igor, and thereby indirectly determines that content. Further, Igor can have the perspective that we attribute only if he himself thinks the (attributed) content of Bela's thought. That is how he has a grip on his canonical concept of the content. Igor's attribution itself names the content. But naming it in this canonical way requires also thinking it. That is the burden of the *Stronger Principle for Canonical Names of Senses*. Understand-

142–144. Cf. also his *Being and Being Known* (Oxford: Oxford University Press, 1999), 245–62. He also makes an objection from an inference involving quantifying into that-clauses. I will not discuss this objection. I believe, however, that the hierarchical view is clearly at no disadvantage in dealing with quantifying in, as long as it is extended to allow for attributions of *de re* attitudes. Frege himself does not handle such cases. But the difficulties that his view faces do not derive from commitment to a hierarchy. I believe that his view needs to be supplemented to handle *de re* constructions and quantifying in. I believe that such supplementation need not essentially affect the structures that we are discussing here.

ing the canonical name requires understanding the customary sense that it names, as well as understanding the customary sense of the iterable construction for forming new canonical names. In our formalization, the iterable construction is "C". In natural language it is the first (or higher) iteration of the subordinate "that-clause" construction. This sense of "C" determines the function from senses to the senses that are canonical concepts of them.

In linguistic mode: Igor could not understand a canonical name for Bela's (presumed) belief content if he did not form the first-level canonical name from an exercised mastery of the expressions that customarily express that content. Here there is a close connection between that-clauses, quotation marks, and canonical sense-names. The advantage of the symbolic canonical names is that their iteration clearly maintains a maximum of extensional contexts. The advantage of the quotation marks and the that-clauses, as denoters of first-level, customary senses, is that they make it clear that understanding the naming device requires a prior understanding of the root named entity. It is a requirement on the symbolic canonical names (at the first level, hence at higher levels) that they be like that-clauses in this respect.

I think it clear that intuition cannot be expected to adjudicate whether the content of Bela's thought is contained in Igor's attribution or, on the contrary, determined by Igor's attribution, *where Igor's way of determining (thinking about) Bela's thought requires also thinking it*. Determination (s-denotation) is a theoretical concept answerable to structural as well as intuitive concerns. I believe that the Fregean view is equally in accord with intuition on the point. The Fregean view is strongly supported by the structural theoretical principles discussed in "Frege and the Hierarchy". It must be evaluated in terms of the power and explanatory value of those principles. (I will return to this issue.) I conclude that this intuitive objection is ineffectual.

A second intuitive objection goes as follows. From (2) and

(3) It is true that Opus 132 is a masterpiece

one can deduce:

(4) Something that Igor believes that Bela believes is true.

If one collapses the levels down to one, one can infer (4) from (2) and (3) by existential generalization. If one retains the levels, the objection goes, one can account for the deductive validity of the inference only through extremely complicated principles.

This objection is incorrect. By utilizing the downward functional structure of canonical names, once a formalization for sense-composition and canonical names is in place, it is a simple matter to produce a simple formalization of the inference. In fact, all that is needed is existential generalization. The key idea is that the embedded canonical (that-clause) name of the customary sense remains available to quantification because of the downward functional structure of canonical sense-names. Thus in the formalization of (2), Igor's attribu-

tion to Bela is specified with the functional expression: C(that Opus 132 is a masterpiece). One can quantify on the whole functional canonical name or on the argument place within it (or indeed onto the positions of the canonical names within the that-clause, which is composed of canonical names of senses).

To make these points a little more explicit: Let angle brackets yield canonical names of customary senses. Let the hat indicate appropriately formulated syntactic composition. Let (2) be formalized

(2a) Believes (I, $<$Bela$>^\wedge<$Believes$>^\wedge$C($<$Opus 132 is a masterpiece$>$)).

Then by existential generalization and exportation:

(EG) $\exists y(y = <$Opus 132 is a masterpiece$>$ & Believes (I, $<$Bela$>^\wedge$ $<$Believes$>^\wedge$C(y))).

One can then use an obvious formalization of (3) to get (4). To capture the inference from (2) and (3) to (4), one need only quantify into the place of the canonical name "that Opus 132 is a masterpiece" (or "$<$Opus 132 is a masterpiece$>$") within the larger canonical name, "C(that Opus 132 is a masterpiece)" in (2a).[20] (For more on existential generalization applied to embedded constructions, see Appendix II.)

I have already explained the cognitive difference between the first-level customary senses and the (indirect) second-level canonical senses that determine them. Between the first and second levels, there are often differences in logical category, which clearly correspond to differences in modes of presentation. For example, the sense of a sentence is different from the sense of a singular term denoting that first-level sense. Moreover, each level of embedded attribution corresponds to different conceptual perspectives and intellectual capacities or levels of sophistication. The argument in "Frege and the Hierarchy" brings out why these distinctions of levels make a logical difference, not just in the relation between the first and second levels, but at each level of attributional embedding.

I believe that collapsing the hierarchy incurs serious costs in a formal representation of attributions of the representational perspectives that consti-

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[20] A more nearly fully formalized language with these properties is set out by Terence Parsons, in "A Quasi-Fregean-Carnapian-Early Kaplanian Semantics", forthcoming in a volume honoring David Kaplan (Oxford: Oxford University Press). Parsons' up-arrow is essentially the same as my "C". We came to the idea independently. I agree with his remark that the hierarchy is "a kind of epiphenomenon of the simple part at the basis of the hierarchy"—with the proviso that one needs, in addition to an understanding of the simple part at the basis, a conception of a functional notion for yielding canonical senses (expressed by his up-arrow and my "C") to understand the cognitive content of the hierarchy. It is important that understanding canonical names at all levels depends on an antecedent *understanding* of the first-level customary senses. Parsons provides a detailed semantics in which he shows how to deal with inferences essentially similar to (2)–(4). He also discusses the argument of "Frege and the Hierarchy" and extends the argument in an illuminating way. I have some doubts about his account of variables relative to an assignment as being canonical names of the assigned objects. I believe, however, that this part of his view is not essential to formalizing relevant inferences.

tute propositional attitudes, and in understanding the relation between sense (or cognitive mode of presentation) and denotation.[21] I want to respond to one attempt to circumvent the argument of "Frege and the Hierarchy".

Christopher Peacocke tries to answer the argument I gave in "Frege and the Hierarchy" to show that Method I incurs serious costs if it is to escape the sense hierarchy.[22] Peacocke's view is that there is only one level of sense. He thinks that the senses of expressions in that-clauses are "redeployed". So the customary senses are not only denoted by obliquely occurring expressions in a that-clause. They are expressed by them as well. This is a version of Method I, one that seeks to avoid not only a hierarchy but even a second level of sense. It seeks to avoid treating indirect sense as distinct from customary sense. The only senses are customary, first-level senses. As indicated earlier, I think that when one reflects on what senses are—ways of thinking about purported referents—this view is one of the least plausible ways of avoiding the hierarchy. It seems to me far less plausible than the view that attempts to stop the hierarchy at two levels of sense. I am doubtful about either way, however.

My objection in "Frege and the Hierarchy" to taking Method I as a way of avoiding the hierarchy centered on difficulties that arise in giving a theory of truth for Method I.[23] Peacocke thinks that my objection rests on an oversight about how to give the truth conditions for natural-language belief contexts using Method I. He notes that although I say that the meta-linguistic semantical account that I gave for Method I does not "give" the senses of the object-

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[21] Daniel R. Boisvert and Christopher M. Lubbers, in "Frege's Commitment to an Infinite Hierarchy of Senses", *Philosophical Papers*, 32 (2003), 31–64, offer an argument from Fregean principles for the hierarchy. Their paper does not make reference to mine. They center directly (and I think illuminatingly) on the functional-compositional structure of senses, whether denoted or expressed. They also make a more explicit textual case than I did that the principles that rule out a hierarchy can be found in Frege. Frege's commitment to the idea that one can recover the structure of thoughts from the structure of expressions expressing them occurs in various places. Cf. e.g. "Logic in Mathematics", *Posthumous Writings*, 207; *Nachgelassene Schriften*, 224. The authors do not make direct use of the substitution principles that I appealed to in an object-language (for Method II) or metalanguage (for Method I), as I did. And they do not discuss the Methods separately. But I explicitly associated the principles that I discuss with the functional-compositional structure of sense that they highlight. In one respect their argument is less general than my argument, or the extension of the argument that I will discuss. It does not apply directly to an attempt to collapse the hierarchy that appeals to only one level of sense. It shows on Fregean principles that if one is committed to two levels of sense, one is committed to the infinite hierarchy. They argue separately against a one-level theory. (I agree with some but not all of their objections to a one-level sense theory.) They also assume, appealing to Davidson's unlearnability argument, that Frege's commitment to the hierarchy of senses is untenable. As I have indicated, I believe that this assumption is very much mistaken.

[22] Cf. Peacocke, "Entitlement, Self-Knowledge, and Conceptual Redeployment", 153–157.

[23] I continue to think that this objection to Method I raises interesting questions about the relation between a truth theory for a language and the structure of that language. I continue to think that the objection is correct. But I believe that there is an objection to taking Method I as a way of avoiding the hierarchy that is more direct. Such an objection works directly off the alleged senses and denotations of the parts, without using any "syntactical" premise about substitutivity. The argument that I gave in the original paper, using a premise about substitutivity in contexts that correspond to oblique contexts in attributions of propositional attitudes, was envisioned as applicable to the metalanguage that gives the truth theory for the Method I. But the last argument I give below can be adapted to apply directly to

language, but only describes them, this should not be surprising. For the truth-theoretic axioms that I give (in Section III of "Frege and the Hierarchy") do not say what the senses of the expressions of the language are. He then goes on to give a truth theory that does give or specify the senses.

This answer rests on a misunderstanding of my objection. I never doubted that one can give or specify the senses of the object-language and carry through a truth theory using those specifications. In fact, under the rubric of "translation" into the metalanguage I specifically discuss a truth theory that specifies senses. My claim (in section III of "Frege and the Hierarchy") was that in a metalanguage that allows substitutivity of co-denoting expressions and that specifies senses by translating expressions of the object-language (in particular ones in oblique contexts that specify senses) into metalinguistic expressions with the same senses, the problems I raised for Method II will recur in the metalanguage. They will recur in a metalanguage capable of providing a systematic semantics for the truth conditions of sentences of a Method I object-language. The problems will recur if one tries to avoid the hierarchy, unless one gives up basic plausible principles about sense and denotation.

The main relevant assumptions invoked in the original article are of three types. One is that the extensional substitution principles needed in a truth theory are maintained. This is the syntactical counterpart of the principle that a sentence's truth-value is fixed by the denotations of its parts. The syntactical counterpart is needed to carry out standard proofs within a truth theory for any ordinary object-language. A second type of principle is that sense determines denotation. For the argument, I needed only an instance of this principle: that sentences with the same sense have the same truth-value. The third type is that the translations are compositional and that principles of functional composition and decomposition of senses are maintained in the metalan-

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an object-language that follows Method I, without making use of any premise about substitutivity that Method I bars. Method I directly incurs a hierarchy on plausible principles. I outline this more direct argument in *Appendix I: Direct Pressure on Method I*. (It is probably advisable to read the Appendix after reading through the main text.)

   Contrary to many who work on these issues, I do not think it at all obvious that Method I is a closer and more natural formalization of natural language than Method II. The issue is how to account for the special sort of "ambiguity" that arises between non-oblique contexts and oblique contexts. Most formalizations of ambiguous terms introduce different terms that correspond to the different "meanings" of a single word-form—just as Method II does. It is true that the type of ambiguity involved in oblique contexts is structural and systematic. It is also true that in oblique contexts, where the customary sense of a word is denoted, the denoted sense plays an essential role in understanding the sense of the occurrence that denotes the customary sense. So the different senses of a word-form in natural language, which are expressed in oblique and non-oblique contexts, are bound together. But Method II as well as Method I incorporates into its account an acknowledgment of this point. In Method II, the acknowledgment lies in the way that understanding higher levels of sense depends on understanding the first level. The issue over the best account of *natural language* ultimately depends on which formalization best accounts for inferences. I venture to predict that Method II will be found to be superior on this score. What I believe with more confidence is this: Method II languages are superior as frameworks for scientific and philosophical theories of thought.

guage. Peacocke does not investigate my claim that the hierarchy will recur in the metalanguage that gives the truth theory for a Method I object-language, unless one gives up one of the principles. (I held that the substitution principle cannot be given up if one is to prove the theorems of the truth theory.) In particular, he does not investigate whether the hierarchy will recur in his own metalanguage. Thus he does not even address the claim of section III of the 1979 paper, much less answer it.

I will sketch some arguments that show how the metalanguage that Peacocke relies upon threatens to lead to the same hierarchical result that I discussed with respect to Method II languages. The first argument that I will give faces a pair of prima facie problems. I will show that the two problems are not fundamental by considering two further arguments. The last of these arguments seems to me to show decisively that my original criticism of the position applies to Peacocke's metalanguage. A fully formalized version of the arguments would be more perspicuous, but would also take up more space. I will provide enough formalization to make them clear.[24]

Let "$< \ldots >$" be a canonical name of the customary sense of the expression that fills in for the dots. (This is Peacocke's terminology.) So "$<\text{Opus132}_L>$" denotes the customary sense of "Opus 132" as it occurs in the Method I, formalizing object-language L. Let "$< \ldots >$ $^\wedge < \ldots . > ^\wedge < \ldots . . >$" denote the result of functionally composing the senses named by the bracketed expressions into a canonical name of the grammatically appropriate complex expression.[25] "$<\text{Opus132}_L> ^\wedge <\text{Masterpiece}_L>$", for example, canonically names the customary sense of the natural-language sentence "Opus 132 is a masterpiece", as those expressions (subscripted by "L") occur in the Method I, formalizing object-language L. Such compositional principles apply to the translations of L's

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[24] I give two semi-formal arguments here. The first argument is an application to Peacocke's language of a simplified variant on my 1979 argument. The second is an application of an extension of my 1979 argument by Parsons, "A Quasi-Fregean-Carnapian-Early Kaplanian Semantics", to singular terms. I think that the argument generalizes further. My original argument focused on sentences within that-clauses. But the basic ideas are the same, as applied to any non-sentential expressions that have sense, not just sentences and singular terms.

An anticipation of the arguments of the present Postscript, pointed out by Tony Anderson, can be found in Leonard Linsky, *Referring* (New York, Humanities Press, 1967), 35. Linsky uses the principle that sentences with the same sense have the same truth-value, as do I. But his other premise is different from the two I use (compositionality and decomposability of sense). For both present dialectical purposes and purposes of clarifying fundamental principles about specification and explanation of thoughts, I prefer my argument. But Linsky's argument is of independent interest.

[25] The notation "$^\wedge$" reads "appropriately grammatically composed with". "$^\wedge$" is a wave of the hand toward what will inevitably be an extremely complex account. The account must specify the various ways in which one puts together names of complex senses of complex expressions, where the complex expressions have components of different syntactical categories. All that I assume here is that canonical names can be formed in such a way that the sense of the name of the sense of a complex expression is a function of the senses of the expressions that are components of the complex expression. The name-forming rules will make use of the syntactical rules governing the expressions with the senses named, and the ordering of the syntactical parts (with their senses) within the syntactical complex.

sentences into the metalanguage. So for example "$<$Opus132$_L>$^∧ $<$Masterpiece$_L>$" also names the sense of the metalanguage sentence "Masterpiece(Opus 132)", since this metalanguage sentence translates the object-language "Opus 132$_L$ is a masterpiece$_L$". And "$<$Masterpiece (Opus132)$>$" names this same sense, by way of the metalinguistic expressions that express it.

According to the view that I oppose,

(a) The customary sense of "Bela believes that Opus 132 is a masterpiece"
    $=<$Bela$_L>$^∧$<$Believes$_L>$^∧$<$Opus 132$_L$ is a masterpiece$_L>$.

I assume (a) for *reductio*. (a) combines the beginning of a functional decomposition of the sense of (1), "Bela believes that Opus 132 is a masterpiece", with the view that in the that-clause of (1), the *customary* sense of "Opus 132 is a masterpiece" is expressed. That is, what is expressed is the customary sense, rather than a further indirect sense.[26]

Now suppose that the sense of "Opus 132 is a masterpiece" (in the natural language) and the sense of its counterpart in the Method I, formalizing object-language are the same as the sense of the sentence that translates it, "Masterpiece(Opus 132)", into the metalanguage. And suppose that the senses of "Bela$_L$" and "Believes$_L$" are the same as their metalinguistic counterparts. So translation is normal but tries to capture Peacocke's redeployment view of the object-language L, according to which there is only one level of sense.

Then

(b) $<$Bela$_L>$^∧$<$Believes$_L>$^∧$<$Opus 132$_L$ is a masterpiece$_L>$=
    $<$Bela$>$^∧$<$Believes$>$^∧$<$Masterpiece(Opus 132)$>$.

(c) $<$Bela $>$ ^∧ $<$ Believes$>$ ^∧ $<$Masterpiece(Opus 132)$>$ =
    $<$Believes(Bela, Masterpiece(Opus 132))$>$.

(c) follows from (b) by the functional compositionality of sense. The sense of the expression within the angle brackets is functionally dependent on the senses expressed by the constituent parts of the expression.

(d) The customary sense of "Bela believes that Opus 132 is a masterpiece" =
    $<$Believes(Bela, Masterpiece(Opus 132))$>$.

(d) follows from (a)–(c) by transitivity of identity.

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[26] Peacocke's account assumes that the that-clause denotes as well as expresses its customary sense. I have mentioned earlier that I believe that this is an unattractive feature of the account—certainly out of keeping with the Fregean view that a sense determines a unique denotation. On his view the sense is a mode of presentation both of a sense and of a truth-value. Or if one rejects truth-values, the sense associated with "Opus 132" is a mode of presentation both of a sense and of a string quartet. Such a view seems to me to lose any plausible connection to the idea that senses are modes of presentation or ways of thinking. This view also loses connection between senses (as thought components) and type-indentification of specific cognitive abilities. The ability to think about a string quartet is different from an ability to think about a sense.

(e) "Bela believes that Opus 132 is a masterpiece" is true in L ⟷ Believes(Bela, Masterpiece(Opus 132)).

(e) follows from (d) by the principle that free-standing sentences with the same sense are materially equivalent. Suppose that "Masterpiece(Opus 132)" is true. Let "S" be any other true sentence in the metalanguage. Then

(f) "Bela believes that Opus 132 is a masterpiece" is true in L ⟷ Believes(Bela, S).

(f) follows from (e) by the extensionality of substitution principles in the metalanguage and substitution of "S" for "Masterpiece(Opus 132)".

Similarly, since "Opus 132" occurs in extensional position in the metalanguage sentence that expresses the sense $<$Bela$>^\wedge<$Believes$>$ $^\wedge$ $<$Masterpiece(Opus 132)$>$, singular terms with the same denotation are interchangeable in sentences that express attributions of belief.

(f) is a *reductio ad absurdum*. What does it reduce to absurdity? The principles of the functional compositionality and decomposability of sense and the principle that if the senses of sentences are the same, they have the same truth-value, seem very plausible. The extensionality of substitution principles in the metalanguage seems necessary for giving a systematic theory of truth.

I see three ways of pinning the absurdity of the conclusion on something other than the assumption that the customary sense of a sentence is expressed in oblique occurrences in object-language that-clauses. I believe that none of these ways will ultimately be satisfactory.

One way is to claim that it is *ungrammatical* to attach "Believes" syntactically to a singular term ("Bela") and a sentence ("Masterpiece(Opus 132)").[27] Such attachment constitutes putting together senses of expressions in a compositional way. In fact, it composes the very senses that Peacocke claims are involved in the belief attribution. It matches the claimed sense of the object-language's that-clause with a sentence in the metalanguage with the same sense. But one could maintain that the compositionality principle is inapplicable at step (c), because the resulting string is ungrammatical. I believe that this claim will not suffice to escape the difficulties, and that it does not go to the heart of the matter.

Frege took sentences to have the same sense and denotation as certain terms—nominalizations of the sentences. We can easily recast the argument so that we use only nominalizations of sentences in that-clause position. We maintain the principle that sentence nominalizations that have the same sense

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[27] There is, in my view, an unclarity about the logical syntax of "believes that *p*" in the object-language if one accepts the view that the sentence in the that-clause both denotes its customary sense and expresses its customary sense. Insofar as it expresses its customary sense, it is a sentence. Insofar as it denotes its customary sense, it is a term. I believe that this unclarity is an aspect of the fundamental problem. I believe that in actual fact the that-clause is a singular term, and "believes" is a relational predicate.

denote the same truth-value—and then carry through the same argument. It seems clear that this doctrine of Frege's is not the key issue in whether or not there is a hierarchy.[28]

Moreover, an argument exactly parallel to (a)–(f) can be applied to the object-language sentence "Byron searched for Ossian". In direct contexts, "Ossian" and "Tlaloc" lack a denotation. "Byron searched for Ossian" can be true while "Byron searched for Tlaloc" is false. Other singular terms can be used to make the point without using ordinary proper names.

The Fregean treatment holds that a term that follows "searched for" that resists coextensional substitution denotes its customary sense. Here there is no change from the grammar of direct contexts to the grammar of indirect contexts. There is a singular term in both cases.

Now consider iterations such as "Browning questioned Byron's search for Ossian" and "Eliot questioned Browning's questioning of Byron's search for Ossian". The analog of the original argument still produces an absurd conclusion ("Bela searched for Ossian" is true in L if and only if Bela searched for T, where "T" is any term coextensive with "Ossian", or coextensive with whatever other term occurs in the position of "Ossian"). This is an absurd consequence that does not at all depend on the ambivalent grammatical position that the view under discussion involves (and forces) in its explanation of the roles of expressions in ordinary that-clauses. So the grammatical issue cannot be fundamental.

So I take the attempt to block (a)–(f) by appeal to the ungrammaticality of the key sentence not to lead anywhere worthwhile.

A second way to attempt to block the argument on behalf of the one-level theory is to allow that there are two relevant "belief" sentences in the metalanguage. One is the one that we have been discussing—"Believes(Bela, Masterpiece(Opus 132))". The other is the same except that instead of a sentence in the second argument place of "Believes", there is a singular expression. This singular expression must both denote and express the customary sense of the sentence, "Masterpiece(Opus 132))". Thus, in the metalanguage, there is a singular term that denotes the customary sense of the sentence, but expresses that same sense. Then one simply denies that "Believes (Bela, Masterpiece(Opus 132))" is true. Only the belief sentence with a singular term in the second argument place can be true. This view tries to block the argument at step (e). It rejects the principle that free-standing, non-indexical grammatical sentences with the same sense have the same truth-

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[28] In "Frege on Truth" (1986) (Ch. 3 above) I maintain that the usual reasons for holding that Frege was mistaken in giving sentences and singular terms a single grammatical category *for some purposes*, are not good ones. I do not think that there are deep philosophical reasons against Frege's view on this point or against his view that sentences and their nominalizations have the same sense. Still, I do not accept either of these views. So I do not ultimately rest my case here on siding with Frege against this complaint. Nevertheless, I think that the complaint is insubstantial and does not go to the heart of the matter.

values. For "Believes (Bela, Masterpiece(Opus 132))" and "Believes (Bela, ST)" have the same sense but different truth-values (where "ST" stands in for a singular term that denotes the sense of "Masterpiece(Opus 132)" and expresses that same sense). I believe that this way of attempting to block the argument is unacceptable. The principle that is rejected is a very basic and plausible one. In fact, this way of attempting to block the argument involves commitment to quite a number of further unattractive consequences. I will return to these and discuss them in the context of a further argument ((a′)–(f′), below).

A third way of attempting to pin the absurdity on something other than the collapse of the hierarchy is to reject an assumption behind step (b) of the argument—the assumption that "Believes" in the metalanguage has the same sense as the object-language "Believes$_L$". One can plausibly argue as follows. It turns out that, in the metalanguage, "Believes" works so that if a person "Believes" (in scare quotes) one truth, he or she believes them all. So what could show more clearly that the metalanguage expression "Believes" does not have the same sense as the object-language "believes" ("Believes$_L$")? Similarly, the absurd results in (e) and (f) simply bring out that the truth theory rests on bad translation from the object-language into the metalanguage. Something is peculiar about either the supposed senses of the sentences or the attempt to match them in translation.

Now I think that this point is correct. But the question is what is leading to this state of affairs? We used translation of the sentence "Opus 132 is a masterpiece" into "Masterpiece(Opus 132)", which seems entirely correct. Those sentences in the respective languages (object-language and metalanguage) express the same sense. And we applied the composition principle with the resources that we had. We need to consider whether good translation is possible, consistent with the principles that we are assuming. This was the dialectical situation that I envisioned in "Frege and the Hierarchy". The argument developed so far is a minor variant on the one given in the original 1979 article. I claimed that if one produces a translation, one will either find the hierarchy in the metalanguage, or run afoul of the relevant Fregean (and otherwise plausible) principles. Let us investigate this matter further. I believe that the problem in translation is just a symptom of the more fundamental disease.

What we need in the metalanguage is a singular term that denotes the customary sense of "Masterpiece(Opus 132)", but does not introduce a further indirect sense. It expresses the sense that it denotes. Perhaps this is the analog in the metalanguage of the way the object-language's that-clauses are supposed to work. This is an idea that we came upon in discussing the first possible difficulty with (a)–(f), the grammatical difficulty. So let us follow it out here.

We continue to take "<Masterpiece(Opus 132)>" to be a singular term in the meta-language that denotes the customary sense of the sentence "Mas-

terpiece(Opus 132)". This singular term is now to be taken as having no further sense: its sense is the same as the sense of the sentence.

This is in itself an absurdity. The sense of the sentence cannot be the same as the sense of a singular term that is about a sense or a way of thinking. The sense of the sentence is a way of thinking about a string quartet and about its being a masterpiece. The sense has components that determine a string quartet and its being a masterpiece. The sense of the relevant singular term, "<Masterpeice(Opus 132)>", is a way of thinking about these ways of thinking.

One needs to bear firmly in mind what senses are. They are perspectival ways in which a referent (if any) is presented to the mind. The problem is not merely that the one-level-of-sense view gives up the principle that a sense determines a referent or denotation. It is that by the nature of sense, a (non-indexical) sense cannot be a single way of presenting such different denotations to a mind.[29] Clearly we are talking about different modes of presentation of different denotations. The idea that the sense of the term that denotes the customary sense of "Masterpiece(Opus 132)" does not introduce a further sense beyond that customary sense of the sentence is, I think, unacceptable. But I will follow out the idea in the context of our principles to show where it leads.

Peacocke does not discuss the senses of the canonical sense-names that he introduces into the metalanguage. But it is obvious that on Fregean theory, they have to have senses. They express different ways of thinking about senses than, say, definite descriptions or other names of the senses that we might whimsically introduce. We should be able to form canonical names of the senses expressed by these first-level canonical names. Thus it would seem that we can regard the expression "C(<Masterpiece(Opus 132)>)" as a canonical name denoting the canonical sense of the metalinguistic expression "<Masterpiece(Opus 132)>". On the one-level anti-hierarchy view, the sense denoted by the first expression will be the same as the sense denoted by the second. (Or perhaps, one would just bar introduction of the first expression.) Of course, we can also produce informal specifications of the sense of "<Masterpiece(Opus 132)>", as follows: the sense of

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[29] In my own view, certain ways of thinking are token applications, or abstractions from token applications. These do not determine their denotations by their nature. They depend on context for their successes. But for any given token application there is at most one denotation or referent. So even these "ways" of thinking are modes of presentation of an object (if any) that are individuated in such a way that a given way of thinking determines at most one denotation. Here way of thinking determines a unique referent, if any, in the functional sense of "determines". This is, I think, the sense of "determines" that is fundamental for understanding Frege and for understanding representation. For more on this, see my "Belief *De Re*", *The Journal of Philosophy*, 74 (1977), 338–362; "Russell's Problem and Intentional Identity" in Tomberlin (ed.), *Agent, Language, and the Structure of the World* (Indianapolis: Hackett, 1983), 79–110; "Vision and Intentional Content" in E. Lepore and R. V. Gulick (eds.), *John Searle and his Critics* (Cambridge, Mass.: Blackwell, 1991); and "Five Theses on *De Re* States and Attitudes", in a forthcoming volume in honor of David Kaplan (Oxford: Oxford University Press).

"<Masterpiece(Opus 132)>". But in my view, these specifications do not have the same sense as the canonical names.

As I envisage a fully formal language, I postulate an initial layer of canonical names for the sense of each primitive expression of the Method II language. Then there will be canonical names, formed with "C", for the sense of those canonical names, and iterations for higher-level senses of the sense of the expression formed with "C", and so on. Thus, "C(<Opus 132>)" is a canonical name of the canonical sense that determines the sense denoted by "<Opus 132>". (Of course, "Opus 132" does not occur as a name in extensional position within the angle brackets.) There are also composition principles for denoting the sense of a complex expression on the basis of the senses of its parts.

If it is to avoid, in the truth-theoretic metalanguage, exactly the same hierarchy of senses that Frege makes use of to account for iterated contexts, the single-level view must claim that for canonical names of senses, the senses of the names are identical with the senses that they denote. This view captures in the metalanguage the way that expressions in that-clauses operate in the object-language—according to the view that collapses the hierarchy to one level (the redeployment view): The expressions both denote and express the expressions' customary senses.

Such a view may seem to make only a small concession to the broadly Russellian idea that names have denotation but no (new) sense—or that sense is identical with denotation. The concession is only for special names, canonical names of senses. For these names, the sense and denotation are the same. The concession appears to be fairly close to Russell's restricted view of "logically proper names," which supposedly name universals that we "grasp" by acquaintance. In fact, this seemingly small concession leads to very serious trouble.

Here is a further argument that is relevant to understanding the difficulties I raised for a metalanguage that specifies senses (or translates object-language expressions into the metalanguage), gives a truth theory for Method I object-languages, and attempts to avoid the hierarchy. The argument is that either the metalanguage is committed to a hierarchy of senses after all, or it falls again into collapsing sense and denotation.

For simplicity, I will carry out the argument on the sense named by "<Opus 132>", rather than on the sense named by "<Masterpiece(Opus 132)>". This will circumvent the grammatical issue discussed earlier. I believe myself to have shown that that issue is not fundamental. But just to keep matters simple, I will avoid it in this argument from the beginning.

In what follows I will use quotation marks rather than canonical names for the expressions of the Method II language. In a fully formal exposition, I would avoid this. But for present purposes, I believe that quotation makes for easier reading. The argument could easily eliminate quotation marks without any substantial change.

Suppose, according to the view that collapses the hierarchy and that makes the concession to Russell:

(a′) The sense of "<Opus 132>" = <Opus 132>.

(a′) says that the sense of the canonical name, "<Opus 132>", which denotes the customary sense of "Opus 132", is not an additional sense. It is the customary sense "redeployed". According to (a′), the sense of the term that translates an unembedded obliquely occurring expression is identical with the (customary) sense denoted by an unembedded obliquely occurring expression. (a′) makes "<Opus 132>" match in the Method II truth-theoretic metalanguage the semantical behavior of the counterpart expression which it translates from the Method I language, or from the obliquely occurring expression in natural language.

As noted earlier, I think that step (a′) is already absurd. A way of thinking about a string quartet cannot be the same as a way of thinking about a way of thinking. Certainly no theory of thought should be committed to this. Methods of individuating cognitive abilities attributed by a scientific or philosophical theory of thought need to be more flexible and fine-grained. In cases of successful representation, they need to individuate ways of thinking in terms of what they are about.

This step, (a′), can be motivated by asking the proponent of a one-level theory of sense to choose the expression in the truth-theoretic Method II metalanguage that most closely translates expressions that formalize in the Method I language the obliquely occurring expressions in the natural language (or the expression in the Method II metalanguage that most closely translates English obliquely occurring expressions).[30] "<Opus 132>" is the term in Peacocke's truth-theoretic, Method II metalanguage that he uses to translate the obliquely occurring expression "Opus 132" in English (or in the counterpart expression in the Method I object-language). I do not know of a better choice. It is a canonical name that "gives" the sense. It does not merely describe it. One must understand the sense denoted (the customary sense) if one is to understand this name of the customary sense. In these respects, the canonical name "<Opus 132>" matches the English obliquely occurring expression and its Method I counterpart expression. Given that this canonical name is the closest translator in the Method II language, and prima facie a good translator, we can ask what the sense of "<Opus 132>" is. According to the view under discussion, the sense of the obliquely occurring expression

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[30] One of the possible "ways out" from the *reductio* about to be developed is to claim that English and a Method I language do not translate into a Method II language. On such a view, no truth-theoretic semantics that relies on translation can be given for English or for a Method II language. One could hold that one can give a semantics for English or Method I, but no translational semantics (i.e. no truth theory that respects Tarski's schema where the right side translates the object-language sentence mentioned on the left side). The *reductio* proof that I am about to give does not purport to defeat such a view directly. But I do not think denial of translation is plausible. At any rate, Peacocke does not take this "way out".

in English is identical with its oblique reference, and there is no need for a further sense. To stay as close to the way that the term that it translates is supposed to work, we say, in (a′), that the sense of "<Opus 132>" is nothing other than the customary sense of "Opus 132".

Then,

(b′) The sense of "<Opus 132> = <Opus 132>" = the sense of "<Opus 132>" $^\wedge$ the sense of "=" $^\wedge$ the sense of "<Opus 132>".

(b′) depends only on the principle that the sense of a whole sentence (the trivial identity sentence) can be functionally decomposed into the senses of its parts.

(c′) The sense of "<Opus 132>" $^\wedge$ the sense of "=" $^\wedge$ the sense of "<Opus 132>" = the sense of "<Opus 132>" $^\wedge$ the sense of "=" $^\wedge$ <Opus 132>.

(c′) follows from (a′) and (b′) by substitutivity of identity (here applied in a context that does not translate or otherwise formalize a natural-language oblique context).[31]

(d′) The sense of "<Opus 132>" $^\wedge$ the sense of "=" $^\wedge$ <Opus 132> = the sense of "<Opus 132> = Opus 132".

(d′) follows from (c′) by functional compositionality of senses. The sense (or sense-proposition or thought content) composed appropriately of the senses of the semantically relevant parts of a sentence is identical with the sense (or sense-proposition or thought content) expressed by the whole sentence.

(e′) The sense of "<Opus 132> = <Opus 132>" = the sense of "<Opus 132> = Opus 132".

(e′) follows from (b′)–(d′) by transitivity of identity.

(f′) <Opus 132> = Opus 132.

(f′) follows from (e′) by the truth of self-identities, propositional calculus, and the principle that free-standing sentences with the same sense have the same truth-value.

The argument is replicable at any higher level of the hierarchy. So it applies to any view that identifies doubly indirect senses and indirect senses.

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[31] A variant of the argument that I am giving is applicable directly to Method I; see Appendix I. The style of argument that I am giving is not restricted to application to a Method II metalanguage. But here I am illustrating a variant of the argument that I gave in "Frege and the Hierarchy" regarding a Method II translational truth theory for English or for a Method I language.

It should be emphasized that we need not have used a canonical name like "<Opus 132>" that makes use of an "internal" occurrence of the name for the customary referent. Any canonical name of the sense of the name for the customary referent would do, as long as it respects the principles governing canonical names for senses.

Analogs apply for other expressions besides names and singular terms. So an analog will apply to senses of predicate expressions.

So if one makes what initially may have seemed to be a small concession to a Russellian conception of canonical sense-names; and if one maintains plausible general principles about sense (especially plausible if one thinks of sense—as one should—as way of thinking), the distinction between sense and denotation for singular terms, indeed for all expressions, collapses across the board. The argument assumes only the functional compositionality and decomposability of senses, substitutivity of identity in contexts that do not translate natural-language oblique contexts, and the principle that the senses of sentences determine their truth-values. This last principle need be applied only to non-indexical sentences standing alone and lacking any analog of oblique contexts.

All of the principles, except for the collapse of the sense and denotation for canonical sense-names, are Fregean principles.[32] All are attractive, and can be independently motivated. I believe that they are very plausible components in any theory of the structure of thought. The principles of functional compositionality and functional decomposability of sense seem to me to be deeply embedded in computational psychological theories of thought. Senses are ways of thinking. Ways of thinking type-identify cognitive capacities. Cognitive psychology and philosophy need to attribute and account for cognitive capacities in ways that correspond to computational abilities, including inferential abilities. In order to do so, they need to specify particular representational abilities that enter into thinking in such a way that they determine the representational perspective and the computational capacities of the full propositional attitude. That is compositionality. Moreover, the attitude perspective and the computational capacities associated with the whole attitude need to be specified in such a way that one can recover specifications of the representational capacities that figure in it. That is decomposability. I think it plausible that any scientific language fit to specify such propositional attitudes for explanatory purposes in

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[32] Frege supports the principles governing sense composition and decomposability in the following passages: Letter to Jourdain, *Philosophical and Mathematical Correspondence*, 79; *Wissenschaftlicher Briefwechsel*, 127; "Compound Thoughts" in *Collected Papers on Mathematics, Logic, and Philosophy*, ed. B. McGuinness (Oxford: Basil Blackwell, 1984), 390; *Kleine Schriften*, 378; O 36. Cf. also letters to Russell in *Philosophical and Mathematical Correspondence*, 149, 157, 158, 163, 165; *Wissenschaftlicher Briefwechsel*, 231, 239, 240, 245, 247; "On Concept and Object", in *Collected Papers*, 193; *Kleine Schriften*, 178; O 205; "On Sense and Denotation", in *Collected Papers*, 163, 166; *Kleine Schriften*, 148–149, 151; O 33, 37. Frege makes clear in his letter to Russell 12/28/1902 that he regards the natural-language shifting of sense and denotation of expressions in attributions of attitudes as an undesirable ambiguity. For a formal language he prefers the introduction of new signs, "though the connection with the corresponding signs in direct speech should be easy to recognize". This view indicates that he would have regarded a Method II language as an ideal language for thought. It also constitutes commitment to the substitution principle. The principle that free-standing sentences that have the same sense (express the same complete thought) have the same truth-value is implied by Frege's numerous discussions of thoughts as the bearers of truth or falsity. Cf. e.g. "The Thought", in *Collected Papers*, 353–354; *Kleine Schriften*, 344–345; O 60–61.

psychology ought to accept the compositionality and decomposability principles.

Substitutivity of identity is necessary in a metalanguage in which a truth-conditional semantics is to be carried out. Moreover, in the argument, substitution is applied only in contexts that do not correspond to oblique contexts.

The idea that free-standing sentences with the same sense have the same truth-value is, I think, also fundamental. Again it is crucial to remember that senses are ways of thinking—representational thought contents. I will discuss this principle at greater length than the others.

First, let us clear away a possible misunderstanding. We are not discussing the senses of free-standing (unapplied) sentences containing indexicals. Such sentences do not even fall under the principle, since the senses—that is, the ways of thinking—expressed by such sentences, apart from application in a context, do not have truth-values. Only when such sentences are supplemented by applications of the context-dependent devices in a context does one have a complete free-standing sentence that expresses a sense (way of thinking, a complete thought) that determines a truth-value.

The principle that free-standing sentences with the same sense have the same truth-value is, I think, an acknowledgment of a fundamental commitment, that is intuitive as well as theoretical. This is the principle that complete *thoughts* expressed by sentences, or by sentence-occurrences, have definite truth-values. (I lay aside non-denoting singular thought components and issues having to do with vagueness.) Complete propositional thoughts *are* truth conditions. They are true or false (or truth-valueless)—but not each, relative to some further parameter. In other words, propositional ways of thinking expressed by "complete" sentences are truth-bearers. I believe that they are the fundamental truth-bearers. Certainly, one of the fundamental intuitive facts about cognitive states is that some of them can be true or false, not merely true relative to one linguistic context, or mode of linguistic expression, and false relative to another.[33]

Suppose that one attempts to block the argument by denying the principle. One thereby disallows the transition from (e′) to (f′). Then one simply accepts that the sense of "<Opus 132> = <Opus 132>" is the same as the sense of "<Opus 132> = Opus 132". One might reason as follows: A sense may either determine itself, or it may determine its standard reference. Truth-value

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[33] This claim does not entail Frege's view that ways of thinking are eternal or context-free. I think that some elements of senses (ways of thinking that happen to be expressed by language) are ineliminably individuated in terms of token representations—what I call "applications". Such representations are individuated in terms of occurrences in time, and are essentially dependent for their representational features and representational success (or failure) on context. On this point, see my "Reference and Proper Names", *The Journal of Philosophy*, 70 (1973), 425–439; "Demonstrative Constructions, Reference, and Truth", *The Journal of Philosophy*, 71 (1974), 205–223; "Belief *De Re*", *The Journal of Philosophy*, *op. cit*.; "Russell's Problem and Intentional Identity", in *Agent, Language, and the Structure of the World*, James E. Tomberlin, ed. (Indianapolis: Hackett Publishing Company, Inc., 1983), pp. 79–110; and "Five Theses on *De Re* States and Attitudes", *op. cit.*

can vary according to which referent the sense has. The referent determined by a sense is relative to a linguistic context.

I see no scientific or philosophical motivation for such a view. Even apart from regarding senses, with Frege, as ways of thinking, the idea that two complete (non-indexical), free-standing sentences with the same sense could have different truth-values is thoroughly unintuitive. It is also, as noted, theoretically unattractive inasmuch as it gives up any straightforward connection between senses and truth conditions.

But when one regards senses, as one should, as ways of thinking, the line of thought is more deeply unacceptable. I have already remarked on the absurdity of step (a′), which is a near-consequence of the view we are discussing. The view also entails that the way of thinking expressed by a singular term can be the same as the way of thinking expressed by a sentence. (The sense of "<Masterpiece(Opus 132)>" is supposed to be the same as the sense of "Masterpiece(Opus 132)".)[34]

The main considerations against this line are philosophical and scientific. Senses are ways of thinking. Where they are expressed by complete free-standing sentences, the ways of thinking are complete, propositional thoughts, or thought contents. Thought contents help type-identify propositional attitudes—psychological states. In an account of propositional attitudes—whether empirical-scientific, logical, or philosophical—canonical specifications of thoughts and thought components that are used in *attributions* of propositional attitudes are the fundamental way in which such attitudes and abilities are specified. Psychological laws or law-like generalizations can be expected to work off of such specifications. For purposes of logic, specification of thought contents and sub-propositional ways of thinking must bear a simple relation to propositional attitude attributions, if logic is to be used as a norm for thinking. So for purposes of understanding thought, as opposed to understanding more practically oriented natural-language ways of communicating about thought, we want a form of propositional-attitude attribution that centers on the nature of the attitudes themselves, and the nature of the propositional representational content that type-identifies the attitudes.

So discourse canonically specifying, and attributing, thoughts and ways of thinking is the most fundamental discourse about the natures of thoughts and ways of thinking. Suppose that one holds that in the most fundamental discourse about the natures of thoughts, the truth-value of a propositional representational thought content can be specified only relative to a linguistic context. Suppose one holds that in the most fundamental discourse about the

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[34] In contrast to Frege, I think that the sense of a sentence (the way of thinking expressed by a sentence) and that of a singular term can never be the same. Frege thought that sentences and sentence-nominalizations could have the same sense. However, I believe that Frege's insistence on distinguishing customary sense from indirect sense suggests that he would have rejected the view that the sense of a sentence and the sense of a name of its customary sense could be the same.

natures of non-propositional ways of thinking that the *representatum* of a way of thinking can be specified only relative to a specification of an expression that expresses the way of thinking. Then one holds that the most basic features—the representational features—of thoughts and ways of thinking can be specified only relative to a linguistic context. Since thoughts and ways of thinking type-identify propositional attitudes, this implies that the most basic features of propositional attitudes can be specified only relative to a linguistic context. Any thought can be attributed at the base of a sequence of iterations. So to block the hierarchy in this way, one must hold this view for all thoughts. Such a view is committed either to the claim that all thought is language-relative (dependent for its most basic features, I would say its very nature, on relation to language) or to the claim that we have a peculiar inability to think about thought as it is. (Cf. note 8.)

There are, of course, philosophers who do maintain that all thought is dependent for its nature on language. This view seems to me to be incompatible with much that is known about both animal psychology and the development of language in human children. Arguments for the view seem to me to have been quite unimpressive.

This philosophical issue seems to me to be not quite as basic, for present purposes, as a scientific one. The view that in attributions of attitudes, one can specify the truth-value of a thought, or the *representatum* of a non-propositional way of thinking, only as being relative to a linguistic context, would make sciences of thought very *peculiarly* language-relative. Even if ways of thinking were all somehow metaphysically language-relative, I see no ground for holding that what they are about and what truth-values they have must be specified, in an empirical scientific theory or a logic, only relative to a particular linguistic expression within the language of the science. This would be an exceptionally strong species of language-relativity that would seem to hold in no other scientific domain. The point of an empirical science or a logic is to specify its subject matter in as general and context-free way as possible. Rejecting the principle that ways of thinking determine a definite *representatum* and complete thoughts have definite truth-values—in favor of the supposed language-relativity—would block normal scientific specification of the subject matters of psychology and logic.

I take the four principles to be fundamental to a logic and psychology of propositional attitudes. Unless one can give grounds for giving up one of the principles as part of these enterprises, one must take the hierarchy to be a fact embedded in the nature of thought. Thus blocking one or more of the principles within a particular language, or for the purposes of accounting for the structure of some possible or actual natural-language locutions, does not suffice to show that the hierarchy is not necessary to a correct understanding of thought. That is how Frege took it. I believe that he was almost surely right.

Let me, however, make a last remark about natural language. One could concede that a sense hierarchy does appear in the metalanguage, but hold that it is not used in accounting for oblique contexts in the object-language, allegedly the natural language. This move seems to me very questionable. It is quite unobvious why one should not be able to translate the hierarchy of sense-names present in the metalanguage back into the object-language, or rather into a natural language. Again, bear in mind that this hierarchy of sense-names is simply a way of specifying ways of thinking or cognitive perspectives. How could one deny that we are capable of thinking in these ways through the medium of natural language? I know of no bar to compositional translation of *these* (non-semantical) expressions between object-language (or natural language) and meta-language.

One can deny principles that lead to a hierarchy. But not only are all the relevant assumptions Fregean. They are well motivated and very plausible. They are especially plausible, it seems to me, as applied to a theory of thought.

Frege's approach to the language of ascription of propositional attitudes, and to thought about propositional attitudes, leaves out a lot. Frege ignores the fact that in many ascriptions we do not care about the exact way that an individual thinks about a subject matter. Sometimes we simply specify the subject matter. In ascriptions we often specify the subject matter of a person's thought in ways that we do not believe correspond at all to the way the person thought about the subject matter. In some of these cases, we "quantify into" that-clause constructions.

Frege's passing over this important and widespread phenomenon derives from his interest in an ideal scientific language for attributing propositional attitudes. An ideal language for this purpose would center on the nature of the attributed attitudes, fully specifying that nature—specifying the way that the relevant person actually thought. He was not interested in natural language ascriptions *per se*. He was especially not interested in attitude ascriptions insofar as the ascriptions are governed by pragmatic or contextual communicative concerns. He was interested in an ideal language for attributing attitudes *in order to specify what the attitudes are*. This involves specifying the representational perspective of the individual with the attitudes. I believe that this is not merely an old-fashioned interest. It remains an interest of scientific psychology and of any common-sense attribution that centers on conveying the nature of the attitude as fully and accurately as possible.

Frege's approach also does not address attribution of *de re* attitudes. Here I believe that his account of thought and sense is inadequate to explain the nature of the attitudes themselves. There are elements of *de re* propositional attitudes that are not timeless thought contents.[35]

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[35] Cf. my "Belief *De Re*" and "Five Theses on *De Re* States and Attitudes", and the Introduction, Part II.

Nevertheless, I believe that Frege's structural approach is, as far as it goes and given its aims, fundamentally on the right track. When one focuses not on linguistic meaning but on thought expressed and attributed by language—and one thinks of thought content as perspective on or way of thinking about a subject matter—the insight and power in Frege's approach tend to emerge. In the context of those aims, I believe that the postulation of a hierarchy of cognitive modes of presentation, or of ways of thinking, or of perspectives that are associated with propositional-attitude attributions, is tenable and probably superior to theoretical alternatives.

The hierarchy is motivated and entailed by basic principles that plausibly apply to thought content, propositional perspectives, ways of thinking. The relevant principles are those of the functional compositionality and decomposability of senses—of representational thought contents—the functional dependence of truth-value on the denotations of constituent linguistic parts (or the *determinata* of constituent representational contents), and the functional (determination) relation between senses and denotations. I believe that in view of the way that senses are formed and denoted through canonical names, the hierarchy is far from the bugbear that most philosophers have presented it as being. In itself, it is simply another example of the functional productivity of language and thought.

The sense hierarchy is, to my mind, particularly interesting in that it provides a new example of how subject matter (here, senses or thought-component concepts) can help individuatively determine the ways we think about a subject matter—even as the way we think about the subject matter also representationally determines the subject matter. The canonical senses or concepts take a cue from Russell: Understanding them requires understanding their denotations. But the sense hierarchy remains Fregean: A cognitive perspective is partial and is to be distinguished from what it is a perspective on.

Appendix I: Direct Pressure on Method I

In "Frege and the Hierarchy" and in the text of this Postscript, I discuss Method I mainly by considering its reliance on Method II for a semantical theory. I show how a hierarchy is induced for a Method II metalanguage for Method I. Here I discuss a more direct argument that suggests that on plausible principles, Method I itself is directly committed to a hierarchy.

It will be recalled that Method I allows systematic ambiguity of terms as between non-oblique contexts and oblique contexts. The point at issue is how far ambiguity extends. One might believe that there are only one or two levels of sense that are associated with embeddings of oblique contexts. In this

Appendix I want to give and discuss an argument for the view that Method I should be directly committed to an infinite hierarchy of senses to account for embedded contexts. The argument will be very similar to the last of the arguments that I give in the text, with one important alteration.

A term like "Opus 132" will be treated by Method I as having a customary sense when it occurs in a non-oblique context—a context like "Opus $132 =$ Opus 132". It will be treated as denoting this customary sense in an oblique context. The question at issue is whether there is a further sense, beyond the customary sense, expressed in ordinary unembedded oblique contexts. A one-level theory of sense gives a negative answer to this question. The argument that I give will be directed against this answer. Substantially the same argument can be given against any theory that stops the level of sense (for a given level of embedding) at a finite level. Thus an adaptation of the argument can be directed against a Method I theory that limits itself to only two levels of sense: the customary sense of an expression, and a further sense expressed by the expression in an oblique context. Such a theory would deny that a further (third) level is needed to deal with what sense is expressed in an embedding of an oblique context within a further oblique context. Similarly, an adaptation of the argument can be directed against a Method I theory that holds that there are only three levels of sense.

To make the argument easy to read, let us introduce a notation for the phrase "the sense of expression E in Context C". Let this expression be abbreviated by the two-place functional expression "S". Let "N" denote the class of non-oblique contexts. For example, "N" would denote a class that contains both singular term positions in the sentence "Opus $132 =$ Opus 132" standing alone (where "$=$" is read as "is identical with"). Let "O_1" denote the class of oblique contexts that are not embedded in a further oblique context. For example, "O_1" would denote a class that contains the position of "Opus 132" (occurring obliquely) in the sentence "Bela believes that Opus 132 is a masterpiece". I use angle brackets ("$< \ldots >$") as I do in the text, except that I include a marker for the context in which the term within the brackets is to be taken as occurring. Thus "$<$Opus132, $O_1>$" is a canonical name for the sense of the expression "Opus 132", as "Opus 132" occurs in an unembedded oblique context. "$<$Opus132, N$>$" is a canonical name for the sense of the expression "Opus 132" as "Opus 132" occurs in a non-oblique context.

Here is the argument:

(1)   S("Opus 132", O_1) = S("Opus 132", N) $=<$Opus 132, $O_1 > =$
      $<$Opus 132, N$>$.

(1) simply records the one-level-of-sense view of the semantics of a Method I language. We will not use the latter two identities in (1) in the argument. I record them simply for clarity.

(2) S("Opus 132 = Opus 132", N) =
     S("Opus 132", N) $^\wedge$ S("=", N) $^\wedge$ S("Opus 132", N).

(2) says, in effect, that the sense of a sentence occurring in a non-oblique context and asserting the self-identity of Opus 132 is decomposable into the senses that the component parts of the sentence express in a non-oblique context.

(3) S("Opus 132", N) $^\wedge$ S(" = ", N) $^\wedge$ S("Opus 132", N) =
     S("Opus 132", N) $^\wedge$ S(" = ", N) $^\wedge$ S("Opus 132", O_1).

(3) follows from (2) and (1) by substitutivity of identity. Note that the substitution does not occur in an oblique context, or in a context that formalizes an oblique context.

Let "a" be a name of the customary sense of "Opus 132", and let "a" be capable of occurring in non-oblique contexts. Let "a" express (in non-oblique contexts) any sense one likes that is compatible with these explanations of its denotation or reference and of its grammatical behavior. Then

(4) a = S("Opus 132", N) = S("Opus 132", O_1).

(4) follows from the explanation of "a" and (1).

(5) S("Opus 132", N) $^\wedge$ S(" = ", N) $^\wedge$ S("Opus 132", O_1) =
     S("Opus 132", N) $^\wedge$ S(" = ", N) $^\wedge$ a.

(5) follows from (4) and (3) by the substitutivity of identity.

(6) S("Opus 132", N) $^\wedge$ S(" = ", N) $^\wedge$ a =
     S("Opus 132 = a", N)

(6) follows from (5) by the compositionality of sense. The sense (or thought) composed appropriately of the senses of the semantically relevant parts of a sentence is identical with the sense (or thought) expressed by the whole sentence.

(7) S("Opus 132 = Opus 132", N) = S("Opus 132 = a", N).

(7) follows from (2)–(6) by the transitivity of identity.

(8) Opus 132 = a.

(8) follows from (7) by the truth of self-identities and the principle that free-standing sentences with the same sense are materially equivalent.

(8) is absurd. It says in effect that Opus 132 is identical with the customary sense of "Opus 132". Since the argument can apply to any expression, and since it can apply to any account that stops at a finite level of senses expressed in successively embedded oblique contexts, it shows that in a Method I language, sense and denotation (or reference) collapse if the principles relied upon in the argument are accepted.

The principal premises used in the argument are again the decomposability of sense, the compositionality of sense, and the principle that free-standing sentences with the same sense have the same truth-value. There are two further background assumptions used in the argument.

One is that it is permissible in the Method I language to use functional terms (containing "S") that specify the senses of expressions relative to contexts, where the specifications are in accord with a given theory's account of what those senses are. Thus one is producing functional specifications of senses with complex singular terms that can occur in non-oblique contexts. I cannot see any ground to reject this assumption.

The other assumption is that it is permissible in the Method I language to introduce a name like "a" that can denote a sense but occur in a non-oblique context. I believe that this assumption is really dispensable in favor of the first assumption. It is just that the argument would require more apparatus to deal with embedding of quotation marks within quotation marks, or some analogous system for denoting expressions that denote expressions. I do not see that there is any principled reason for rejecting either of the assumptions.

I have claimed that the argument just given can be adapted to show that the attempt to stop the hierarchy at a finite level, in accounts of embedded oblique contexts, will on the relevant principles collapse the distinction between sense and denotation. Since these matters are complex, I will illustrate how the argument works at one higher level—against a view of Method I languages according to which customary sense and indirect sense are distinguished, but indirect sense is identical with doubly indirect sense. That is, the following argument reduces to absurdity the view that in a Method I language, there are only two levels of sense. It reduces to absurdity the view that the sense of an expression occurring obliquely in an embedded oblique context is the same as its sense in an unembedded oblique context (though distinct from its customary sense, which is denoted in an unembedded oblique context).

I will use iterated quotation marks that embed quotation marks, but I think it clear that these can be dispensed with, without affecting the argument. Here is the argument:

(1′) $S(\text{"Opus 132"}, O_2) = S(\text{"Opus 132"}, O_1).$

(1a′) $S(\text{"Opus 132"}, N) \neq S(\text{"Opus 132"}, O_1).$

(2′) $S(\text{"}S(\text{"Opus 132"}, O_1) = S(\text{"Opus 132"}, O_1)\text{"}, N) =$
        $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge S(\text{"Opus 132"}, O_1).$

(2′) follows from the decomposability of the sense of a sentence asserting that the indirect sense of "Opus 132" is self-identical.

(3′) $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge S(\text{"Opus 132"}, O_1) =$
       $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge S(\text{"Opus 132"}, O_2).$

(3′) follows from (2′) and (1′) by substitutivity of identity.

Let "b" be a name of the sense of "Opus 132", as it occurs in non-embedded oblique contexts. That is, "b" names the indirect sense of "Opus 132". It names $S(\text{"Opus 132"}, O_1)$. Let "b" be capable of occurring in non-oblique contexts. Let "b" express (in non-oblique contexts) any sense one likes that is compatible with these explanations of its denotation or reference and of its grammatical behavior. Then

(4′) $b = S(\text{"Opus 132"}, O_1) = S(\text{"Opus 132"}, O_2)$.

(4′) follows from the explanation of "b" together with (1′).

(5′) $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge S(\text{"Opus 132"}, O_2) =$
   $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge b$.

(5′) follows from (3′) and (4′) by substitutivity of identity.

In an unembedded oblique context "Opus 132" denotes only its customary sense and expresses its indirect sense ($S(\text{"Opus 132"}, O_1)$). Let us introduce a term "B" that, in a non-oblique context, expresses the sense that "Opus 132" expresses in an unembedded oblique context and denotes the sense (the customary sense of "Opus 132") that "Opus 132" denotes in that same unembedded oblique context. So "B" denotes $S(\text{"Opus 132"}, N)$. That is, "B" has the same semantical characteristics in a non-oblique context that "Opus 132" does in an unembedded oblique context. Then

(6′) $S(\text{"Opus 132"}, O_1) \wedge S(\text{" = "}, N) \wedge b =$
   $S(\text{"B = b"}, N)$.

(6′) follows from (5′) by the compositionality of sense and the explanation of the term "B".

(7′) $S(\text{"}S(\text{"Opus 132"}, O_1) = S(\text{"Opus 132"}, O_1)\text{"}, N) = S(\text{"B = b"}, N)$.

(7′) follows from (2′)–(6′) by the transitivity of identity.

(8′) $B = b$.

(8′) follows from (7′) by the principle that sentences (standing alone) that have the same sense also have the same truth-value.

(8′) contradicts (1a′). Thus again, on plausible principles, a Method I language will collapse the hierarchy altogether if it identifies an expression's sense with its denotation at any level of embedding of oblique contexts.

The plausible principles are the decomposability of sense, the compositionality of sense, and the principle that sentences (standing alone) with the same sense have the same truth-value.

As in the first argument in this Appendix, there is one additional assumption: We can introduce into Method I languages names or descriptions that in non-oblique contexts express the senses and denote the denotations that other expressions express and denote in oblique contexts. I see nothing in

the conception of Method I languages, or in the conception of Method I languages that I am criticizing, that prevents one from introducing such expressions.

What these arguments show is that the substitutivity of coextensive expressions that differentiates Method II from Method I languages, and which makes Method II languages superior as a basis for giving a truth theory, is not fundamental to yielding the arguments for the hierarchy. More basic is having expressions that can appear in non-oblique positions but which mimic the semantical behavior of expressions that appear in that-clauses. They must denote and (after the first-level) express the senses of expressions that appear in that-clauses. In a Method I language in which a term can occur either non-obliquely or obliquely at various levels of embedding, the argument simply requires that there be other expressions that have the same semantical characteristics—have the same sense and denotation—as occurrences of terms in oblique position. A scientific language that has any chance of describing the structure of sense or thought content as expressed in natural language must have such terms.

Thus on plausible principles, Method I languages generate a hierarchy just as surely as Method II languages do.[36]

Appendix II: Existential Generalization on Embedded Positions

In the main text of the Postscript, I discussed existential generalization on

(2) Igor believes that Bela believes that Opus 132 is a masterpiece.

(2) entails

(2Exp) There is something, namely that Opus 132 is a masterpiece, that Igor believes that Bela believes,

which entails

(2EG) There is something that Igor believes that Bela believes.

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[36] In unpublished work Nathan Salmon has derived the hierarchy from different principles applicable within a Method I language. For example, he uses the shift principle that the indirect reference of an expression is the customary sense of the expression together with principles governing the identity between the reference of an embedded expression and the indirect reference of an expression to which a shift operator expression (such as the "that" in a that-clause, or "the thought that") is applied but at one lower level of embedding. Thus the key principles are those governing shifts within a Method I language. I believe that Frege clearly accepted the principles that Salmon relies upon, and that these further proofs of the hierarchy are interesting both in capturing Frege's thinking and in themselves. But I believe (and Salmon agrees) that the principles of functional composition and decomposition that I have centered on are more basic in Frege's thinking. And I believe that Frege obviously preferred a Method II language as an ideal language for attributing propositional attitudes. (Cf. again the letter to Russell 12/28/1902.)

I formalized (2) as

(2a) Believes (I, $<$Bela$>^\wedge<$Believes$>^\wedge$C($<$Opus 132 is a masterpiece$>$)).

I formalized (2Exp) as

(EXP) $(\exists y)(y = <$Opus 132 is a masterpiece$>$ & Believes(Igor, $<$Bela$>^\wedge<$Believes$>^\wedge$ C(y))).

(2a) also formally entails

(EG) $(\exists y)($Believes(Igor, $<$Bela$>^\wedge<$Believes$>^\wedge$ C(y))).

I believe that (EG) formalizes (2EG).
    It is worth nothing that (2a) also formally entails

(EG′) $(\exists z)($Believes(Igor, $<$Bela$>^\wedge<$Believes$>^\wedge$ z)).

Here it may be tempting to raise an objection to the account that I have given. One might be tempted to hold that in view of the fact that (EG) and (EG′) both follow from (2a), it is a consequence of (2a)'s formalizing (2) that I am committed to (2)'s entailing that Igor believes Bela believes at least two things.

I believe that this objection is mistaken. This Appendix will explain why. The larger point of the explanation is to elicit a better understanding of the hierarchical account, and to explore its relation to English. I believe that (EG) and not (EG′) formalizes (2EG).

As noted, (EG) and (EG′) are both entailed by (2a). It is worth noting that there are yet *further existential generalizations in the offing*. In the first place, there will be more existential generalizations (EG″), (EG‴)...with further embeddings ("Arnold believes that Igor believes that Bela believes that Opus 132 is a masterpiece"), and further iterations of "C" in the formalizations. In the second place, there are positions in the structured root proposition-name that denote senses that are components of the denoted proposition or thought content (e.g. the position of "$<$Opus 132$>$" in the structured "$<$Opus 132 Masterpiece$>$", which really has the form "$<$Opus 132$>^\wedge<$Masterpiece$>$"). From these positions one will have even more quantifications that follow from the initial formalization (2a).

I think that the first of the quantifications, (EG), is what formalizes the English quantification

(2EG) There is something that Igor believes that Bela believes.

Only (EG) captures the inference to (2EG) from

(2) Igor believes that Bela believes that Opus 132 is a masterpiece.

I believe that I can say why (EG) and not (EG′) formalizes (2EG). I will begin by explaining why (EG) and not (EG′) tracks the inference from (2) to (2EG). I will come back later to discussing the interpretation of (EG′).

The reason begins with this observation: (2EG) follows from (2) inasmuch as Igor is characterized in (2) as believing that Bela believes a certain *proposition* or *thought content* (namely, that Opus 132 is a masterpiece). That is, we got (2EG) by first reasoning from (2) to

(2Exp) There is something, namely that Opus 132 is a masterpiece, that Igor believes that Bela believes,

and then dropping the conjunct that consists in "namely that Opus 132 is a masterpiece".

Perhaps it is possible for Igor to be so odd that he believes that Bela believes things other than propositions. Perhaps—although this is at best questionable—Igor could believe that Bela believes a fire engine, or a stone. Perhaps (again questionably) Igor could believe that Bela believes the sense of a singular term, a sense which is not a thought content or proposition. Bela cannot *in fact* believe a fire engine or the sense of a singular term. What might be disputed is whether Igor could believe he can. It would be easier on us all if we could just declare that no one can believe that anyone believes something other than a proposition or a thought content. Perhaps then there would be fewer entailments to worry about! But let us proceed on the assumption that it is possible for someone erroneously to believe that someone believes something other than a proposition or thought content.

However, the only thing that it is guaranteed that Igor believes that Bela believes, *given the truth of the sentence (2)* , is a proposition or thought content—namely, the thought content that Opus 132 is a masterpiece. There are *not* two things that Igor believes Bela believes in virtue of the truth of (2).

I think that the same is true of (2a). (2a) does *not* entail anything that commits us to saying that Igor believes that Bela has another belief besides the belief that Opus 132 is a masterpiece. The only relevant term that denotes a proposition or thought content that Igor could believe Bela believes in my formalization (2a) is "<Opus 132 is a masterpiece>" – *not* "C(<Opus 132 is a masterpiece>)". The latter does not denote a proposition or thought content. It denotes the sense of the canonical name "< Opus 132 is a masterpiece >". (This latter canonical name denotes the customary sense of "Opus 132 is a masterpiece".) So, even if we grant that it is possible, it would be *very* odd of Igor to believe that a non-proposition is believed by Bela. And it is certainly not guaranteed or implied by (2) or (2a) that he believes that Bela believes any such thing. So if we export "C(<Opus 132 is a masterpiece>)", we are not exporting a name of some proposition that Igor believes Bela believes. It is wrong to think that the canonical name (or designator) names anything that Igor believes that Bela believes. I will come back to how we should read such exportations.

Both in English grammar and in formalizations of English, one should keep track of the level of entity (and within sense levels, the type of entity—e.g. sense of a singular term, sense of a sentence, and so on) that is being

quantified over. One has to do some of this anyway to distinguish quantifying into that-clauses (in effect, all the way down to the bottom level of reference or denotation) from quantifying onto them (quantifying over a sense or mode of presentation). Thus, in a full theory of quantification, one should mark the variable (here "y") that "C" applies to as a variable that goes with the name of a propositional thought content—as distinguished from the name of a customary sense of a name, and as distinguished from the name of a customary sense of a predicate.[37]

We have the following situation: (2EG) follows from (2). (2a) formalizes (2). Both (EG) and (EG′) follow from (2a). So both (EG) and (EG′) should follow from (2). But (EG′) does not generalize on a second belief of Igor's, a belief over and above the belief that Opus 132 is a masterpiece (contrary to the tempting objection that we began with). So generalization on the position of "C(<Opus 132 is a masterpiece>)" in (2a) is not what is going on in the English (2EG). (2EG) is *not* true in virtue of a generalization on "C(<Opus 132 is a masterpiece>)". For this would take the relevant singular basis for the existential generalization to be the sense of a singular term, not a thought content—the sense of a sentence.

I stipulate that Igor does not believe that Bela believes the sense of a singular term. Igor attributes only propositional beliefs to Bela.

Another way of putting all this is that we cannot get to a formalization of (2EG) by exporting "C(<Opus 132 is a masterpiece>)" from (2a) to get

(EXP′) $(\exists z)(z = C(<$Opus 132 is a masterpiece$>)$ & Believes(Igor, Bela$>^{\wedge} <$ Believes $>^{\wedge} z))$,

taking this to be a formalization of

(Weirdo) There is something, namely the canonical sense of the name "that Opus 132 is a masterpiece", that Igor believes that Bela believes

and then dropping the first conjunct of the existential generalization to get

(EG′) $(\exists z)($Believes(Igor, $<$Bela$>^{\wedge} <$Believes$>^{\wedge} z))$,

taking this to formalize

(2EG) There is something that Igor believes that Bela believes.

There is no analogy here to first exporting "that Opus 132 is a masterpiece" from (2), then formalizing this exportation by (EXP), and then simplifying (EXP) to (EG).

In view of the fact that the "z" in (EG′) does not trace back to the name of a proposition, it is not made true by anything Igor believes that Bela believes. Moreover, (Weirdo) is *not a correct reading* of (EG′). For "the canonical

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[37] Of course, Church's Logic of Sense and Denotation has this feature.

sense of the name 'that Opus 132 is a masterpiece' " is a definite description and "C(<Opus 132 is a masterpiece>)" is a canonical name or canonical designator—not a definite description. Our canonical names represent specifically oblique occurrences of expressions in natural language, most prominently such occurrences in ordinary or in embedded that-clauses. "C" specifically marks levels of embedding in canonical names of modes of presentation or representational content.

Just to fix this last point, let us consider formalizations of

(2Weird) Igor believes that Bela believes the fire engine

(2Weirdo) Igor believes that Bela believes the canonical sense of the name "that Opus 132 is a masterpiece".

I believe that one can take each of these as having a quantifying-in reading and an oblique occurrence reading, if they have any readings at all. Let us focus on the oblique readings. The propositions that Igor believes are *expressed* by the sentences

(2Weird -) Bela believes the fire engine

(2Weirdo -) Bela believes the canonical sense of "that Opus 132 is a masterpiece",

where the direct objects occur obliquely. Suppose that the terms in direct-object position denote their customary senses. Suppose that we allow first-level canonical names to formalize those occurrences and to denote the customary senses:

<the fire engine>

<the canonical sense of "that Opus 132 is a masterpiece">.

Then in formalizations of (2Weird) and (2Weirdo) we must introduce canonical names that denote the senses of these canonical names, as those senses are expressed in (2Weird -) and (2Weirdo -). Thus we formalize (2Weird) and (2Weirdo) as

(2Weird - f) Believes(Igor, <Bela>$^\wedge$<Believes>$^\wedge$C(<the fire engine>))

(2Weirdo - f) Believes(Igor, <Bela>$^\wedge$<Believes>$^\wedge$C(<the canonical sense of "that Opus 132 is a masterpiece">)).

I believe that these formalizations convincingly suggest two things. One is that if one keeps track of whether a canonical name formed with "C" derives from a root canonical name of a complete propositional thought content, or a root canonical name of a sense other than such a content, one will never get formalizations in embedded cases that confuse the two, or that are inspecific as between the two.

The other thing suggested by the formalizations is that canonical names enter initially as formalizations of occurrences of *other* expressions of the

natural language that occur obliquely. On Method II, which we are following, *no canonical name can itself occur obliquely*. The first-level canonical names (which I have been formalizing with angle brackets) will formalize all root weird occurrences. (Cf. the formalizations of (2Weird -) and (2Weirdo -).) "C" comes into play *only* in formalizations of embeddings—usually embedded that-clauses, but perhaps also embedded attributions of Weird beliefs. I believe that *in no case* is it true that such formalizations allow a canonical name which includes "C" to formalize a root direct object of an embedded occurrence of "believe"—whether the object denotes a proposition or not. Thus (EG′), understood as quantifying onto a place that had contained a canonical name that includes "C" does not formalize any sentence like (2Weird) or (2Weirdo). I conjecture that wide-scope quantifications that formalize natural-language quantifications on the direct object of the final or root direct object always leave the "C" (possibly a string of iterated "C"'s) in place (i.e. not exported), as (EG) does. I will soon discuss how to read back into English formalizations that do export canonical names that include "C".

There are surely further things to be said of

(2EG) There is something that Igor believes that Bela believes.

(2EG) should follow even from Igor's believing that Bela believes a fire engine or believes the sense of a singular term. Our formalizations follow this course. For example, (EG), which does follow from (2Weird) and (2Weirdo), is a formalization of (2EG). As I said, I believe that the English grammar as well as the formal theory should leave a trace of the fact that here (2EG) derives from an attribution by Igor of a non-propositional belief to Bela.

The key point for present purposes is that (EG′) should not be the formalization of the generalization (2EG) that follows even from (2Weird) and (2Weirdo). It is (EG) that is the relevant formalization of the generalization (2EG) that follows from (2Weird) and (2 Weirdo).

So an issue has become: what *other* English sentence might be formalized by (EG′)? Let us first consider where the existential generalization in (EG′) came from. Let us think about how we would read the exportation of "C(<Opus 132 is a masterpiece>)" from (2a). The formalized exportation is

(EXP′) $(\exists z)(z = C(<\text{Opus} \ 132 \ \text{is a masterpiece}>) \ \& \ \text{Believes(Igor,} \ <\text{Bela}>^{\wedge} <\text{Believes}>^{\wedge} z))$.

This can be glossed in English:

(Engl-Exp′) There is a (canonical indirect-sense-level) way of thinking about the thought that Opus 132 is a masterpiece that Igor's belief utilizes in attributing that thought (that Opus 132 is a masterpiece) to Bela as a belief.

One could get from this reading down to this English gloss on (EG′):

(Engl-EG′) There is a way of thinking about a thought that Igor's belief utilizes in attributing a thought to Bela as a belief.

Or, more briefly,

(Engl′-EG′) There is something that Igor's belief utilizes in attributing a belief to Bela.

Could this be the English sentence that (EG′) formalizes?

One might protest that these glosses have "extra" words—"way of thinking", "thought", "utilizes", "attributing", that have no counterparts in the formalizing sentences (EG) or (EG′). The fact that there isn't a *smooth* reading in English of (EG′) is not, I think, a difficulty.

I believe that the situation with the English in this case is broadly analogous (but perhaps not ultimately as deeply interesting) as the problem about expressing second-order quantification in English. From the natural formalization of "Sally found Bill"—F(s, b)—, one can infer

($\exists$F)F(s, b).

One can put this in English as something like

Sally bore some relation to Bill.

This sounds more "English", as well as more genteel, than "Sally F'd Bill, for some F".

One might reply, "But then the verb is 'bore' ", or "But then the relation is bore". Well, yes. But it does not follow that we cannot do enough meta-talk about how to understand the English that it is clear that we are understanding words like "bore" as syncategorematic, or as indicators of the second-order quantification. Surely one can express the second-order quantification in English. So it seems that we should be able to explain an understanding of the English according to which we have a second-order quantification over a two-place relation (in "Bill bore some relation to Sally") rather than a first-order quantification over an argument or *relatum* in a three-place relation bore. I think that one *can* engage in such meta-talk and that English can express the second-order locution, though it does not do so smoothly on its surface.

In the case of the "excess" quantifications in the embedded that-clause cases (of which (Engl′-EG′) is an example), it is not a matter of second-order quantification. But I think getting the readings in English of the formalizations will force "extra" words into the English in an analogous way. Here the syncategorematic words are ultimately "utilizes" and "attributes".

I conjecture that any view that both treats that-clauses as *singular terms* with *structure* and treats that-clauses in natural language as having *oblique* positions (i.e. any view that has any chance of being correct as an account of English) will throw up quantifications that cannot be smoothly read back into English, but which are true and are credibly expressible in English.